POLÍTICA Y ESTABILIDAD MONETARIA EN EL PERÚ

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9

Income distribution and endogenous dollarization

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In this chapter we combine portfolio decisions of individuals and invoicing decisions of firms into a general equilibrium cash-in-advance monetary model to explain the pattern of dollarization across types of goods. This framework provides a theoretical link between asset and transaction dollarization. We find that transaction dollarization depends positively on asset dollarization. The exact relationship between transaction and asset dollarization is shaped by the income distribution. Furthermore, for partial asset dollarization, luxury goods, those associated to high-income customers, are endogenously priced in foreign currency, while high priority goods, those associated to low-income customers, are priced in domestic currency. When dollarization is partial, asset dollarization is always higher than transaction dollarization.

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9.1 Introduction

A history of monetary mismanagement and episodes of hyperinflation, especially during the eighties and in some cases during the nineties, transformed the monetary systems of many emerging economies into bimonetary systems. Argentina, Bolivia, Peru, Uruguay, Turkey, and more recently Russia are among those economies in which the domestic currency has been partially replaced in its functions as a reserve of value, medium of payment and unit of account by a foreign currency, usually the US Dollar, a phenomenon known in the literature as dollarization.[1] The process of dollarization has a well documented pattern: usually the foreign currency is used first as a reserve of value, then as a medium of payment in some transactions, particularly big transactions, and finally as unit of account.

Nowadays, even after several years of low and stable inflation, dollarization levels remain high in these countries. However, the levels of asset dollarization, measured by the proportion of deposits or bank loans in dollars, tend to be much higher than the levels of transaction dollarization, usually measured with the most liquid component of deposits.[2] Not only is dollarization different across assets and transactions, but also amongst types of transactions. It is observed that the dollar seems to dominate transactions associated to consumption of high-income customers, while transactions and prices of goods associated to consumption of low-income customers, like necessity goods, tend to be in domestic currency. This is independent of whether the goods are tradable or not tradable or the size of the transaction.[3]

The distinction between different types of dollarization and the links amongst them have crucial implications for monetary policy and macroeconomic performance. As Ize and Levy-Yeyati (2003) point out, while asset dollarization could seriously affect the transmission mechanism of monetary policy and make the financial system

[1] In this chapter we distinguish among three different concepts of dollarization: transaction dollarization (TD), the substitution of domestic currency as medium of payment; asset dollarization (AD), the substitution of domestic currency as reserve of value; and price dollarization (PD), the substitution of domestic currency as unit of account.

[2] Honohan and Shi (2001) provide indirect evidence of low levels of price dollarization in countries with high levels of asset dollarization. They measure price dollarization by the short-run level of pass-through of the exchange rate.

[3] For instance in Peru, private schools set prices in different currencies depending on their location: in rich neighborhoods prices are in dollars, while in poor ones prices are in soles (the domestic currency). Moreover, small transactions like haircuts are charged in dollars in some beauty shops located in rich neighborhoods, whereas big transactions, like real estate are priced in soles in poor areas.

more vulnerable to exchange rate fluctuations, it is price dollarization that ultimately determines the effectiveness of monetary policy. In an economy where most of the prices are set in foreign currency, prices become perfectly indexed to the exchange rate, eliminating the short-run effects of monetary policy. Moreover, understanding the pattern of price and transaction dollarization and its relationship with asset dollarization can be useful to guide policy makers in their attempts to implement policies aimed at reducing dollarization.

In this chapter, we provide a theory of endogenous dollarization of assets and prices that explains the pattern of dollarization across types of goods and the links amongst them. The model combines dollarization decisions of individuals and invoicing decisions of firms into a general equilibrium cash-in-advance monetary model. In modeling individuals' dollarization decisions we follow Chatterjee and Corbae (1992) in that a fixed cost of accessing financial markets determines endogenously the market participation of agents. In our setting, individuals have to pay a fixed cost to dollarize their assets, therefore only those agents with levels of income high enough to pay the cost dollarize. This simple assumption generates the result that not all agents in the economy dollarize, but only those who can afford it, thereby establishing a link between asset dollarization and the income distribution.

We then extend the invoicing decision problem of firms to the case of non-homothetic preferences.[4] We show that with non-homothetic preferences and some degree of asset dollarization, some firms are willing to set prices in foreign currency because their profits become a convex function of the exchange rate. In this case, expected profits are an increasing function of the exchange rate variability.[5] Moreover, with non-homothetic preferences, we are able to introduce endogenous heterogeneity in the demand for goods, where demand and price elasticity depend on the income distribution, unlike the representative agent framework. A key feature of the model is that individuals consume a different number of goods, and consequently each firm does not sell its goods to every individual in the economy, but only to those who can afford it. In contrast, with homothetic preferences the income distribution does not affect the

[4] One of the first works in invoicing decision theory is Klemperer and Meyer (1986), who discuss the decision between Cournot and Bertrand oligopoly competition. Other papers, such as Giovannini (1988), Donnenfeld and Zilcha (1991), Johnson and Pick (1997) and Bacchetta and van Wincoop (2005), study the decision of pricing in the exporter's or the importer's currency in international trade.

[5] With homothetic preferences, only in the case of increasing marginal costs do some firms find it optimal to set prices in foreign currency. With constant marginal costs, firms always choose to set prices in domestic currency (see Bacchetta and van Wincoop, 2005).

demand for goods. In this case, the aggregate demand for each good depends only on the average income and the relative prices.

The general equilibrium shows that asset dollarization causes price dollarization and that income distribution plays an important role in explaining the pattern of price dollarization across type of goods. In particular, we find that for income distributions that show some degree of inequality, necessity goods, those associated with the consumption of low-income customers, are priced in domestic currency (*pesos* hereafter), whereas luxury goods are priced in dollars. Moreover, the model shows that asset dollarization is larger than transaction dollarization, and the gap between them is increasing in the degree of inequality.

Our model is related to the works of Sturzenegger (1997) and Ize and Parrado (2002), which analyze endogenous dollarization decisions but in different frameworks. Sturzenegger (1997) uses an endogenous cash-in-advance model to assess the welfare implications of currency substitution. In his framework, the size of the transaction is the key feature in explaining the pattern of dollarization. Agents decide the currency in which to trade by comparing the fixed cost implied by trading in dollars with the inflation tax, i.e. the cost of trading in pesos. As the inflation tax is proportional to the value of the transaction, expensive goods are endogenously traded in foreign currency since the benefit of trading with dollars (avoiding the inflation tax) exceeds its cost.

This approach, however, does not explain why small transactions associated with high-income customers are made in foreign currency. We instead consider the most important element in determining dollarization patterns to be the interaction between the level of income of customers and the optimal strategies of firms in setting prices. This interaction implies that price dollarization is not independent from asset dollarization, as in Sturzenegger (1997).

On the other hand, Ize and Parrado (2002) use a representative agent general equilibrium model to analyze the interaction between price dollarization, asset dollarization, and monetary policy. In their model, asset and price dollarization are endogenous decisions based on minimum variance portfolios. Hence, both asset and price dollarization respond to the variance of real exchange rate and inflation, but price dollarization also responds to monetary policy and to the nature of the shocks. They conclude that price dollarization is lower when monetary policy is set optimally to maximize the welfare of domestic agents, and when shocks are idiosyncratic. Deviations from optimal monetary policy promote price dollarization, in particular, when shocks are correlated with foreign shocks. They also find that asset and price dollarization are positively correlated but they can differ significantly. More precisely, high levels of asset dollarization may coexist with low levels of price dollarization. However, because they have a representative agent model, they are not able to explain the pattern of dollarization across types of goods.

This chapter fills some gaps in the existing literature. In particular, our model explains, in a simple fashion, the pattern of price and asset dollarization across type of goods and agents, and also provides a theoretical link between types of dollarization. Moreover, our results suggest that policy makers aiming at reducing dollarization should focus on reducing asset dollarization, since price and transaction dedollarization will endogenously follow. The model is also able to explain why high levels of asset dollarization may coexist with low levels of price dollarization.

The rest of the chapter is organized as follows. In Section 9.2 we present the general equilibrium cash-in-advance model without considering dollarization decisions. In Section 9.3, we discuss in detail the dollarization decision of individuals, the invoicing decisions of firms, and the general equilibrium with dollarization. In Section 9.4 we discuss the link between asset and price dollarization. Section 9.5 concludes. The proofs of our main results are presented in the appendix.

9.2 Basic environment

The economy is populated by a continuum of infinitely lived agents that enjoy utility from consuming a set of differentiated consumption goods. There are no savings decisions in the economy, and in every period agents consume all their income.[6] Agents are heterogeneous in their asset holdings. There are two types of assets in the economy: currency and shares of a mutual fund; and a production factor, capital, that exists in a fixed amount and does not depreciate. The mutual fund owns all the firms in the economy and the capital stock. This fund acts as an implicit insurance mechanism, pooling the profits generated by the firms and the flow of income generated by the stock of capital.

The distribution of shares, in turn, determines the distribution of income across agents. This distribution is time invariant, therefore there is no social mobility: an individual born with a certain amount of shares would always consume according to the income associated to these shares. The demand for money is determined by a cash-

[6] We refrain from modeling saving decisions since our focus is on the cross-sectional differences in dollarization decisions amongst agents. For this analysis, intertemporal effects are not necessary.

in-advance constraint that limits the amount of goods that individuals can purchase to the amount of their money holdings. The central bank can change the amount of money in the economy through transfers of currency to individuals.

In addition, firms transform capital into a variety of consumption goods using a linear technology. Each firm produces only one type of consumption good and sets prices to maximize monopolistic rents. There exists only one currency in which income and prices are denominated, the peso. All the dividends are distributed in pesos, and there is no uncertainty.

In this basic setup, the timing is as follows: at the beginning of every period agents receive income distributed from the mutual fund that corresponds to the profits generated by firms and the rent of capital from the previous period. Then, the central bank transfers money to households, firms set prices, and production and consumption take place. Finally, profits and the rent of capital are transferred to the mutual fund.

We adopt the following notation: nominal variables are represented by capital letters and real variables, by lower case letters. Indices i and j correspond to individual's and firm's variables, respectively; variables without index are aggregates. Also, variables with a prime superscript ($'$) denote next period values.

9.2.1 Goods and preferences

There is a discrete number J of goods, indexed by $j = \{0, 1, \ldots, J\}$, which is endogenously determined by the income distribution and the structure of preferences. Preferences are non-homothetic, so income changes the marginal utility over goods.[7] In particular, we follow the setting of Matsuyama (2002), where individuals can consume only one unit of each good, and goods are not substitutes. As a result, richer individuals would consume a higher number of goods in equilibrium.

All individuals have the same utility function, given by:

$$U_i = \sum_{j=1}^{J} \left(\prod_{r=1}^{j} x_{r,i} \right) + \varepsilon x_{0,i} \tag{9.1}$$

[7] A non-homothetic utility function is defined as a set of preferences that exhibits nonlinear Engel's curves, i.e. the expenditure in good i increases nonlinearly with income. With homothetic preferences, for some normalization of the utility function, doubling quantities doubles utility, so Engel's curves are straight lines that go through the origin, and expenditure in good i increases linearly with income.

where $x_{r,i}$ is an indicator function, with $x_{r,i} = 1$ if good $r > 1$ is consumed and $x_{r,i} = 0$ if it is not, and $x_{0,i}$ is leisure.

This function has the property that individuals do not benefit from consuming good h if $x_{r,i} = 0$ for some $r < h$. This implies that the individuals consume good h, only if they can also consume all the other goods with indices lower than h. In other words, individuals have a well-defined priority over the set of goods in their shopping list: goods with a lower index are necessity goods, while those with a higher index are luxury goods. Also, it is assumed that ε is small enough such that $\varepsilon P_j / P_0 < 1$ for every j. This condition guarantees that the consumption of any affordable good would always be preferable to the consumption of leisure.

The budget constraint of an individual i is given by:

$$M_i' + \sum_{j=0}^{J} P_j x_{j,i} \leq M_i + P y_i + T_i \,, \tag{9.2}$$

where M_i represents the beginning of period money holdings, M_i' denotes the money holdings at the beginning of the next period, Py_i is the income transfer from the mutual fund, T_i is a transfer from the central bank, P_j is the nominal price of consumption good j, and P is the price deflator of aggregate output. Individuals also face the following cash-in-advance (CIA) constraint, which generates their demand for money:

$$\sum_{j=0}^{J} P_j x_{j,i} \leq M_i + T_i \,. \tag{9.3}$$

The CIA limits the amount of consumption of individuals to their money holdings: initial money balances plus the transfer from the central bank. Note that because the utility of future consumption is zero, the CIA constraint is always binding, i.e. individuals find it optimal to spend all their cash holdings at every period. Thus, the demand for money of individual i is:

$$M_i^d = P y_i \,. \tag{9.4}$$

Aggregating across individual money demand functions, we can express the equilibrium condition in the money market as:

$$P y = M' \,, \tag{9.5}$$

where M' is the money supply defined as:

$$M' = M + T \,. \tag{9.6}$$

Because of the well-defined priority over the goods, the individual's consumption problem can be simplified as follows: choose q (a discrete variable), the number of consumption goods, and x_0 (a continuous variable), the amount of leisure good to consume, to maximize:

$$U_i = q_i + \varepsilon x_{0,i} \,. \tag{9.7}$$

Thus, the consumer problem can be stated as individuals purchasing as many goods as possible from the top of their shopping list and spending the remainder of their cash holdings on the leisure good. Then, the demand of individual i takes the following form:

$$\begin{aligned} I_q &\leq M_i + T_i < I_{q+1} \\ x_{i,0} &= (M_i + T_i - I_q)/P_0 \end{aligned}$$

where $I_q = \sum_{j=1}^{q} P_j$ can be interpreted as the minimum level of cash holdings that allows individual i to consume q goods. An important feature of these preferences is that additional cash holdings translate into an additional demand for the next good on the shopping list, but only when it passes a threshold. Otherwise, the leisure good is consumed. Then, the indirect utility can be expressed as:

$$V_i = q_i + \bar{\varepsilon}(M_i + T_i - I_q) \,, \tag{9.8}$$

where $\bar{\varepsilon} = \varepsilon / P_0$.

9.2.2 Income distribution and aggregate demand

The mutual fund aggregates the profits of the monopolistic firms, the capital stock and the sales of the leisure good. Individuals own shares, θ, in this mutual fund. At the end of each period, the mutual fund transfers to the individuals the income obtained from the three different sources. The income distribution is described by the cumulative density function of the shares $F(\theta)$ and it has support over the interval $[\,\underline{\theta}, \bar{\theta}\,]$, with $0 < \underline{\theta} < \bar{\theta} < \infty$ and

$$\int_{\underline{\theta}}^{\bar{\theta}} \theta dF(\theta) = 1 \,. \tag{9.9}$$

Individual's i income at the end of the period is given by:

$$Py_i = \theta_i(\Pi + Rk + P_0 x_0) \tag{9.10}$$

where Π, k and R are the total nominal profits, the capital endowment and the rental price of capital, respectively. From the cash in advance constraint, the implicit demand for money of individual i is given by:

$$\frac{M_i^{d\prime}}{P} = y_i = \theta_i y\,, \qquad \text{where} \qquad y = \int_{\underline{\theta}}^{\bar{\theta}} y(\theta)dF(\theta)\,. \tag{9.11}$$

The individual cash holdings during the period are equal to $M_i + T_i$, the initial cash holdings plus the transfer from the central bank. Then, $F(Z/M)$ is the fraction of individuals whose cash holdings are lower than or equal to Z.

The share θ is the only source of heterogeneity across individuals. Since only the individuals with cash holdings higher than $I_j = \sum_{h=1}^{j} P_h$ purchase good j, and no individual purchases more than one unit of each good, the aggregate demand for good j is equal to the mass of individuals whose cash holdings are higher than $I_j = I_{j-1} + P_j$:

$$x_j^d = 1 - F\left(\frac{I_j}{M}\right)\,. \tag{9.12}$$

The non-homothecity of the preferences gives special features to this demand function. As in Matsuyama (2002), the demand is bounded from above by one and it depends on the income distribution. Moreover, because the marginal propensity to spend on a good varies with income, higher index goods will be purchased only by high-income customers, while lower index goods will be purchased by almost all of them. Furthermore, a decline in the price of good h does not affect the demand for good $j < h$ ($\partial x_j^d/\partial P_h = 0$), while it generally increases the demand for good $j > h$ ($\partial x_j^d/\partial P_h > 0$). Therefore, there exists demand complementarity from a lower indexed good to a higher indexed good, but not the other way around.

9.2.3 Firms

There is a discrete number of firms J, each one monopolistically producing a variety of good j. All of them have the same linear technology in capital: $x_j = \lambda_k k_j$. Firms choose prices optimally to maximize profits:

$$\Pi_j = P_j x_j^d - Rk_j = \left(P_j - \frac{R}{\lambda_k}\right)\left[1 - F\left(\frac{I_j}{M}\right)\right]\,. \tag{9.13}$$

From the first order condition, prices must satisfy:[8]

$$\frac{P_j}{M} = \frac{1 - F(I_j/M)}{F'(I_j/M)} + \frac{R/M}{\lambda_k}. \tag{9.14}$$

Prices of monopolistic goods are proportional to the quantity of money M and R. Note that this is a recursive problem: that is, P_j is determined after the determination of prices $P_{j-1}, P_{j-2}, \ldots$. We shall assume that there exists perfect information about the distribution of income, so all the prices can be determined simultaneously, as each monopolist knows the price of the other products, given the distribution of income.

For any $z \in [\underline{\theta}, \bar{\theta}]$, define:

$$G(z) = \frac{1 - F(z)}{F'(z)} + \frac{R/M}{\lambda_k} \qquad \text{and} \qquad \gamma(z) = -F''(z)\frac{1 - F(z)}{F'(z)^2}. \tag{9.15}$$

The expression for $G(z)$ is useful to determinate the optimal price for good j that satisfies $P_j/M = G((I_{j-1} + P_j)/M)$. On the other hand, $\gamma(z)$ is a local measure of the concavity of the income distribution function and it is related to the income inequality. When $\gamma(z) > 0$ [resp. < 0], the income distribution is concave [resp. convex] around z. Given two distributions of θ with the same support $[\underline{\theta}, \bar{\theta}]$, named A and B, if A has higher $\gamma(z)$ than B for every z, then B first-order stochastically dominates A, and the income distribution of B is less unequal than that of A.

The form of the sequence of prices $\{P_j\}$ is described by the following proposition:

Proposition 1. *The second order profit maximizing condition implies that, in order to have bounded prices, it is necessary that $\gamma(z) < 2$. Moreover, for $z = I_j/M$ prices would be locally decreasing if $\gamma(z) \in (-\infty, 1)$, locally increasing for $\gamma(z) \in (1, 2)$, and constant for $\gamma(z) = 1$.*

From Proposition 1 we conclude that the sequence of prices is shaped by the income distribution: it would be locally increasing for any convex income distribution.

[8] This condition is equivalent to:

$$\frac{P_j - R/\lambda_k}{P_j} = \frac{1}{\eta(P_j)},$$

where $\eta(P_j)$ is the price elasticity of demand. This condition states that the "Lerner index", i.e. the relation between the profit margin (price minus marginal cost) and the price is equal to the inverse of the price elasticity of demand. When marginal costs are zero ($R = 0$), the price that satisfies this condition is such that the price elasticity of demand is equal to 1.

Whilst for a concave distribution, prices would decrease for relatively low concavity ($0 < \gamma(z) < 1$), increase when concavity is high ($1 < \gamma(z) < 2$) and be constant for $\gamma(z) = 1$. Additionally, the second order condition implies that $G'(z) < 1$, which guarantees that $P_j/M = G(\,(I_{j-1} + P_j)/M\,)$ has a solution for any $j < J$.

Higher income inequality gives a more concave income distribution. This implies a more inelastic demand curve when moving from the top to the bottom of the shopping basket, which increases the monopolistic power of firms. When $\gamma(z) > 1$, the goods with higher indices become more inelastic, and the monopolistic firms can charge a higher price for them. On the other hand, when $\gamma(z) < 1$ the higher indexed goods become more elastic, so the monopolistic firms charge a decreasing sequence of prices.

P_J, the price of the last good, is a special case. It must satisfy:

$$\frac{R}{\lambda_k} < P_J = \bar{M} - I_{J-1} \leq MG\left(\frac{I_{J-1} + P_J}{M}\right), \tag{9.16}$$

where $\bar{M}$ is the cash holdings associated to the upper bound $\bar{\theta}$. The last firm J charges a price lower or equal to the optimum, such that only the richest individuals can buy the product. Therefore, this condition determines the number of firms J and it depends on the shape of the income distribution. Moreover, the number of firms is bounded because individual income is bounded.

The relationship between J and income inequality has an inverted U-shape form: the number of goods increases for low levels of inequality and decreases for high inequality.[9] Income inequality has two effects in J. On one hand, higher income inequality increases the monopolistic power of the firms, then prices are higher and the number of goods is smaller. On the other hand, income inequality increases the dispersion of income, which increases the number of goods because demand becomes more heterogeneous.[10] Therefore, for low [resp. high] levels of inequality

[9] This result is consistent with Matsuyama (2002) in which a mass consumption society, an economy with high diversity of goods, is formed for intermediate levels of inequality. In contrast, in Foellmi and Zweimuller (2004) higher income inequality increases the number of goods. This is because their model features perfect competition, so higher income inequality only increases the diversity of goods.

[10] Consider a perfectly egalitarian distribution, such that all the individuals have the same income. In this case, the first firm would charge a price equal to the total individual income and the number of goods produced would be one. On the other hand, consider a very unequal income distribution such that a small mass of the population has an extremely high income. In this case, the first firm would charge a price equal to the total individual income of the rich, and the poor would consume only the leisure good. In these two extreme cases the number of goods produced is one.

the latter [resp. former] effect dominates and the number of goods is increasing [resp. decreasing].

The price of capital R is determined from the market clearing condition

$$k^d(R) \equiv \sum_{j=1}^{J} k_j^d(R) \leq k\,, \qquad \text{where} \qquad k_j^d(R) = \frac{x_j(R)}{\lambda_k} \tag{9.17}$$

is the demand of capital of the firm j. From this market clearing and the profit maximization conditions it is possible to see that R is proportional to M and that the firm's capital demands depend positively on R. We assume that the stock of capital is high enough so that $k^d(R) < k$. In this case the price of capital will be $R = 0$. This assumption greatly simplifies the algebra without changing the results. Firms do not use all the capital stock even with zero cost, because under monopolistic competition they find it optimal to limit the quantity produced below the maximum capacity.

Finally, leisure (good 0) is sold directly by the mutual fund and it has zero production costs. We assume that its price is proportional to the average price of monopolistic goods, $P_0 = \delta \sum_{j=1}^{J} P_j/J$. Since the price charged by monopolistic firms is proportional to the money supply, P_0 is proportional as well.

9.2.4 Basic equilibrium

The equilibrium in this economy is defined as a number of firms and goods J, a set of consumption bundles $\{x(\theta)_j\}$ for $j = 0, 1, \ldots, J$ and $\theta \in [\underline{\theta}, \bar{\theta}]$, and a set of prices $\{P_j\}$ and R, such that all individuals maximize their utility subject to their budget constraints, all firms maximize profits, and the goods, factors and money markets clear. Moreover, we define the steady state in this economy by an equilibrium where the cash holdings distribution is invariant, thus real variables will be constant and the nominal variables will grow at a constant rate. The economy deviates from the steady state if the central bank implements a monetary policy through transfers that temporarily change the distribution of cash holdings. The effects of monetary policy are summarized by the following proposition:

Proposition 2. *In this economy, money is neutral if monetary injections are made through transfers proportional to the initial money holdings and if all the firms adjust prices. Otherwise money is not neutral.*

As the demand functions depend on the distribution of cash holdings, monetary policy does not change the real variables when transfers are proportional to the initial money holdings and all firms adjust prices. Any other form of transfer will change the distribution of cash holdings, affecting the demand of some goods, and therefore the equilibrium of real variables will change.

Furthermore, given the non-homothecity of preferences, the way that transfers are implemented affects relative prices among goods differently. When transfers are more than proportional to initial money holdings for the lower income individuals, the demand and the price for low-index goods increase, and because of the asymmetric demand complementarity, the demand of high-index goods decreases. In other words, this form of monetary policy expands the demand for low-index goods but contracts the demand for high-index goods. On the other hand, transfers that are more than proportional for the higher income individuals change the prices of high-index goods. However, this does not affect the demand of low-index goods.[11]

9.3 Dollarization

We now extend the basic model by introducing a second currency, the "dollar", that circulates simultaneously with the peso. Thus, agents have an extra decision to make: choose the currency denomination of assets and prices.[12]

The price of the dollar in terms of pesos, the exchange rate, is an exogenous random variable and represents the only source of uncertainty in the model. The percentage change of the exchange rate, s, is distributed with a cumulative distribution function $H(s)$, with support $[\underline{s}, \bar{s}]$ for $-1 < \underline{s} \leq 0 \leq \bar{s} < \infty$, and expected value:

$$s^e = \int_{\underline{s}}^{\bar{s}} s dH(s)\,, \tag{9.18}$$

which is assumed to be positive. As we have defined the exchange rate as the price of foreign currency in terms of domestic currency, $s > 0$ [resp. $s < 0$] represents a depreciation [resp. an appreciation] of the domestic currency.

[11] These results contrast with the case of homothetic preferences, in which all the individuals have the same consumption basket, so monetary transfers affect all prices in the economy in the same way, independently of the way these transfers are implemented.

[12] More precisely, the decision of dollarization refers to denominating in dollars the flow of cash generated by shares, which in this model coincides with individuals' cash holdings.

Dollarization, the choice of the currency denomination, is costless only for prices, but not for assets. To dollarize their assets, individuals have to sign a stage contingent contract with the central bank at a fixed real cost c.[13] In this contract, the central bank commits to transfer an amount of pesos, T_i, contingent on the realization of the depreciation of the exchange rate s, and proportional to the nominal value of the flow generated by their assets net of the cost, $M_i - Pc$.[14] Therefore, T_i is defined as follows:

$$T_i = s(M_i - Pc)\,. \tag{9.19}$$

Agents make dollarization decisions before observing the realization of s, taking as given the income and depreciation rate distributions. For simplicity, we assume that in this equilibrium monetary policy takes place only through dollarization contracts with individuals. Thus, money supply changes only through T_i.

The timing is as follows: at the beginning of every period, agents receive income transfers from the mutual fund that corresponds to the profits generated by firms and rents from the previous period. After agents have received their income, individuals decide whether or not to dollarize their assets and firms decide to set prices either in pesos or in dollars, given the income and exchange rate distributions. Then, nature draws a realization of the exchange rate. Given the set of prices, the realization of the exchange rate and the income distribution, production and consumption take place. Finally, profits and the rental payments of capital are transferred to the mutual fund.

9.3.1 Individuals

The motivation of individuals to dollarize their assets is not just that of protecting their purchasing power against bad realizations of the exchange rate, but also of taking advantage of the expected capital gain of holding foreign currency. As expected utility is linear in expected income, all individuals have the incentive to dollarize when the exchange rate is expected to depreciate. However, since this is costly, not every one can afford it.

[13] The introduction of this cost attempts to capture the fact that not every individual in a society has access to financial instruments to protect financial wealth against inflation or devaluations. On the other hand, we consider this cost as fixed, but we acknowledge that it may depend on the level of dollarization, as economies with a history of high dollarization may develop cheaper ways to dollarize.

[14] We assume that the revenues generated by the central bank through the dollarization contracts are transferred to the mutual fund at the end of every period. This assumption avoids the central bank accumulating real resources through time.

Individuals decide to dollarize their income by comparing their expected utility levels with and without dollarization. The indirect utility function of individual i can be written as the sum of the number of goods she can afford to consume, q_i, plus the amount spent on leisure, $M_i - \bar{I}_q$, weighted by $\bar{\varepsilon}$. We use $\bar{I}_q$ to denote the ex-post expenditure in domestic currency of consuming q goods. Note that $\bar{I}_q$ is a contingent variable, its value depends on the realization of the depreciation rate given that $q - m$ goods have prices in dollars: $\bar{I}_q = I_q + (1+s)(I_q - I_m)$ for $q > m$ and $\bar{I}_q = I_q$, otherwise, where m is the number of goods with prices in pesos. Thus, the indirect utility function can be expressed as $V_i = q_i + \bar{\varepsilon}(M_i - \bar{I}_{q_i})$ so the corresponding levels of utility with and without dollarization are given by:

$$V_i^D = q_i^D + \bar{\varepsilon}(M_i - Pc + T_i - \bar{I}_{q_i^D}) , \tag{9.20}$$

$$V_i^{ND} = q_i^{ND} + \bar{\varepsilon}(M_i - \bar{I}_{q_i^{ND}}) . \tag{9.21}$$

With dollarization, the level of cash holdings decreases with the payment of the fixed cost, Pc, but increases [resp. decreases] with the transfer from the central bank, T_i, in states of the nature where the currency depreciates [resp. appreciates]. Without dollarization, cash-holdings are not affected by the exchange rate, but as $q - m$ goods are priced in dollars, the exchange rate affects the number of goods that individuals can afford. Therefore, we consider utility under dollarization and non-dollarization as state contingent variables: dollarization takes place only when $\mathbb{E}\{V_i^D - V_i^{ND}\} \geq 0$.

Proposition 3. *Only individuals with cash holdings $M_i \geq M_n \equiv (1 + s^e)Pc/s^e$ choose to dollarize their income.*

From Proposition 3, relatively rich individuals, with cash-holdings higher than M_n, dollarize their income. The mass of individuals who do not choose dollarization is given by those with cash-holdings $M_i < M_n$. Therefore, we can define as n the mass of individuals who choose not to dollarize as:[15]

$$n = F\left(\frac{(1 + s^e)Pc}{s^e M}\right) \equiv F\left(\frac{M_n}{M}\right) . \tag{9.22}$$

[15] The individual dollarization threshold can also be expressed in terms of shares holdings, θ_i, as

$$\theta_n = \frac{c/y}{s^e/(1 + s^e)} ,$$

where c/y is the cost of dollarizing assets as a proportion of the mean income and $s^e/(1 + s^e)$ is an index of the expected depreciation rate. Notice that θ_n is increasing in c/y and decreasing in s^e.

The dollarization decision is independent of m, the number of goods with prices in domestic currency, a result that comes from the linearity of preferences on q and x_0. As utility is piecewise linear in income, we can write the differences of expected utility under dollarization and non dollarization only as a function of expected income.[16] Thus, under our preference setting, the only moment of the distribution of s that is relevant for dollarization decisions of the individuals is its mean, s^e.

The decision of dollarization of individuals changes the ex-post distribution of money holdings. Those individuals who decide to dollarize will increase [resp. decrease] their money holdings relative to those individuals who do not dollarize in states of the world where the exchange rate depreciates [resp. appreciates].

Let M_i^e be the ex-post money holdings of individual i. The ex-post money holdings distribution function, conditional on a mass n of individuals with assets in pesos and on a realization of the exchange rate above its mean, can be written as:

$$F\left(\frac{M_i^e}{M}\,\middle|\, n, s > s^e\right) = \begin{cases} F\left(\frac{M_i^e}{M}\right) & \text{if } M_i^e \leq M_n\,, \\ F\left(\frac{M_n}{M}\right) & \text{if } M_n < M_i^e \leq \frac{1+s}{1+s^e} M_n\,, \\ F\left(\frac{M_i^e}{(1+s)M} + \frac{Pc}{M}\right) & \text{otherwise}\,, \end{cases} \tag{9.23}$$

whereas conditional on a realization of depreciation rate below its mean:

$$F\left(\frac{M_i^e}{M}\,\middle|\, n, s < s^e\right) = \begin{cases} F\left(\frac{M_i^e}{M}\right) & \text{if } M_i^e \leq \frac{1+s}{1+s^e} M_n\,, \\ F\left(\frac{M_i^e}{M}\right) + F\left(\frac{M_i^e}{(1+s)M} + \frac{Pc}{M}\right) - n & \text{if } \frac{1+s}{1+s^e} M_n < M_i^e \leq M_n\,, \\ F\left(\frac{M_i^e}{(1+s)M} + \frac{Pc}{M}\right) & \text{otherwise}\,. \end{cases} \tag{9.24}$$

The conditional distribution function of cash holdings is contingent on the realization of the exchange rate. For realizations of s larger than s^e, this function has a piecewise form, with a flat segment at $n = F(M_n/M)$ and is flatter than the initial distribution for $M_i^e > (1+s)M_n/(1+s^e)$, as shown in Figure 9.1. On the other hand, when the realization of s is smaller than s^e, the distribution displays a kink that depends on the realization of the exchange rate. When s is further away from s^e, the kink in the

[16] The expected utility function is the sum of the expected income and a function that is common under dollarization and non-dollarization. This common function depends on the distribution of prices, the degree of price dollarization and the distribution of the exchange rate. See the proof of Proposition 3.

distribution is also further from n and the new distribution is steeper than the initial one for $M_i^e > (1+s)M_n/(1+s^e)$, as shown in Figure 9.2. Note that, following an appreciation, agents who decided to dollarize transfer money holdings to the central bank ($T_i < 0$), therefore the mass of agents with lower money holdings increases. Thus, the distribution becomes steeper in this range.

9.3.2 Firms

Firms have to decide in which currency to set their prices. Because the exchange rate affects the elasticity of demand, the demand for good j and the profit for this firm will depend not only on the price of the good, but also on s, i.e. $x^d(P,s)$ and $\Pi(P,s)$. Let P^* be the price of the goods in dollars expressed in domestic currency at the initial exchange rate, then $\Pi^D((1+s)P^*,s)$ and $\Pi^{ND}(P,s)$ are the nominal profits expressed in domestic currency when the price is set in pesos and in dollars, respectively.

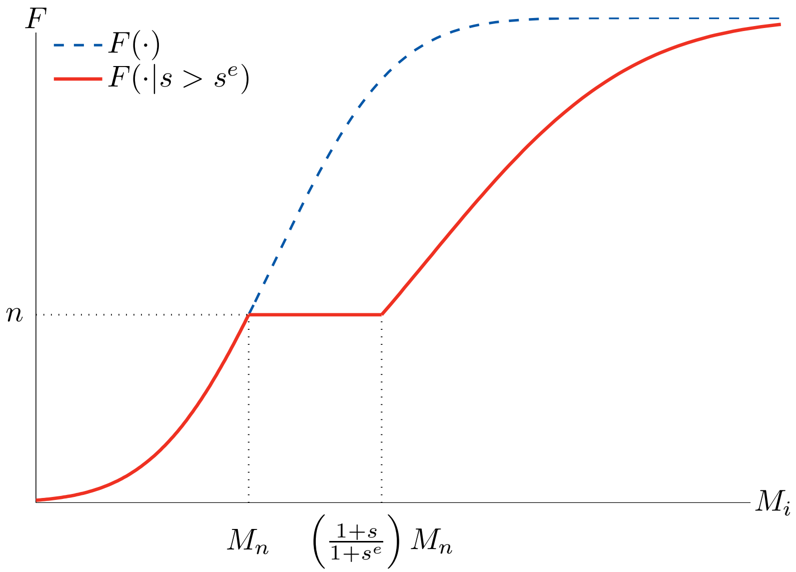
Under perfect certainty about s, the currency price-setting problem becomes irrelevant, because $P = (1+s^e)P^*$. When introducing uncertainty, firms compare the expected profits under the two price setting options. A firm will set the price in dollars if $\mathbb{E}\{\Pi^D - \Pi^{ND}\} > 0$. The problem is not straightforward because the expected value of profits depends on how the exchange rate affects the demand of the goods. To overcome this difficulty, we follow a methodology similar to Bacchetta and van Wincoop (2005), and focus on uncertainty near $s = s^e$, the expected exchange rate, and on "small" amount of risk, i.e. the variance of s, σ^2, tends to zero. Derivations for a general case, when σ^2 is not small, are available upon request.

Lemma 1. *For a twice differentiable profit function, around $s = s^e$ and $\sigma^2 = 0$,*

$$\frac{\partial \mathbb{E}\left\{\Pi^D\left((1+s^e)\,P^*,s^e\right) - \Pi^{ND}\left(P,s^e\right)\right\}}{\partial \sigma^2} \simeq \frac{P}{1+s^e}\left[\Pi_{12}(P,s^e) + \frac{1}{2}\left(\frac{P}{1+s^e}\right)\Pi_{11}(P,s^e)\right].$$

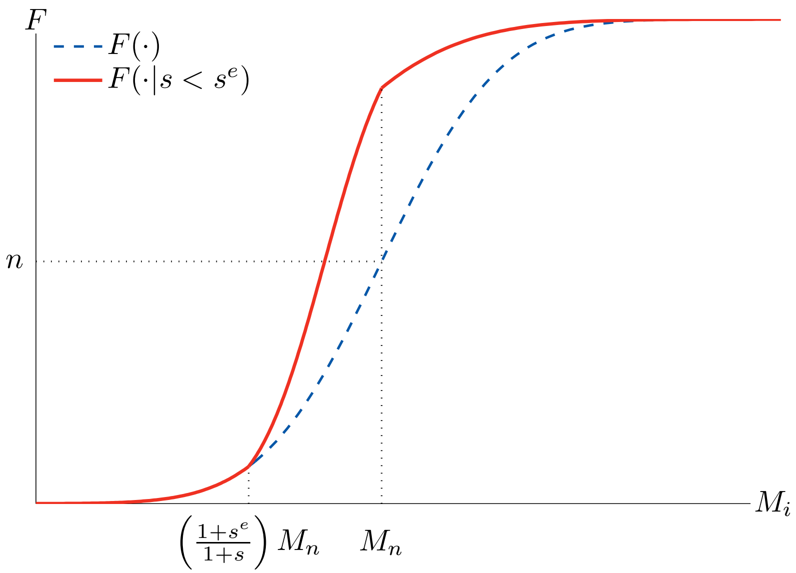
This expression depends on the second order derivatives of the profit function, evaluated at the price in pesos and the exchange rate under perfect certainty. Π_{12} and Π_{11} are the derivatives of the marginal profits with respect to the exchange rate and the price, respectively. The former is related to the marginal benefits of setting the price in dollars (the increase in marginal profits due to a depreciation), while the latter is related to the marginal cost (the decrease in marginal profits due to an increase in the price). Interestingly, none of these expressions depend on P^*.

Figure 9.1 *Cash-holding distribution for $s > s^e$*



Source: Authors' own elaboration.

Figure 9.2 *Cash-holding distribution for $s < s^e$*



Source: Authors' own elaboration.

The currency price setting depends on the sign of the expression in Lemma 1. Prices are set in dollars when the difference of expected profits, $\mathbb{E}\{\Pi^D - \Pi^{ND}\}$, is a convex function of the exchange rate, i.e. when $\Pi_{12}(P, s^e) > -\frac{1}{2}\Pi_{11}(P, s^e)P/(1 + s^e)$. From the profit maximization second order condition we have that $\Pi_{11}(P, s^e) < 0$, then the right hand side of the inequality is positive. To set the price in dollars it is necessary that the marginal benefits of dollarization (Π_{12}) are positive and higher than the marginal costs $-\frac{1}{2}\Pi_{11}(P, s^e)P/(1 + s^e)$. Therefore, for any case where $\Pi_{12} \leq 0$ it will not be optimal to set the price in dollars. Such are the cases when a depreciation reduces the demand of the good or does not affect it at all.

We now use Lemma 1 to analyze the case with non-homothetic preferences and income inequality.[17] For $\sigma^2 \to 0$, given the fraction n of the population with assets in pesos and the other $1 - n$ with assets in dollars, the profits have the following form:

$$\Pi_j\left(P_j, s, I_{j-1}\right) = \begin{cases} \left(P_j - \frac{R}{\lambda_k}\right)\left[1 - F\left(\frac{I_{j-1}+P_j}{M}\right)\right] & \text{if } I_{j-1} + P_j < \theta_n M\,, \\ \left(P_j - \frac{R}{\lambda_k}\right)\left[1 - F\left(\frac{1}{1+s}\frac{I_{j-1}+P_j}{M} + \frac{c}{y}\right)\right] & \text{otherwise,} \end{cases} \tag{9.25}$$

where $\frac{c}{y} = \theta_n\left(\frac{s^e}{1+s^e}\right)$ is the proportion of the fixed cost to the average income.

The demand of the good j depends on its price (P_j), the depreciation rate (s) and the price of the goods with index lower than j (I_{j-1}). The depreciation rate affects the demand of goods only when the total expenditure in goods (I_j) passes the threshold $\theta_n M$. This is the case when all the individuals that demand the good have their assets in dollars.

Proposition 4. *Given that a mass n of individuals maintain their assets in pesos, and given the distribution of s, with expected value s^e and variance $\sigma^2 \to 0$, there exists a threshold level of expenditure*

$$I_{\bar{m}} = M(\,\theta_n - G(\theta_n)\,)\,,$$

such that $\mathbb{E}\left\{\Pi^{ND}(I_{j-1})\right\} > \mathbb{E}\left\{\Pi^{D}(I_{j-1})\right\}$ *for* $I_{j-1} < I_{\bar{m}}$. *Additionally, if*

$$\gamma\left(\frac{I_{j-1} + P_j}{M}\right) < \bar{\gamma} \equiv \left[1 - \frac{1}{2}\left(\frac{P_j}{I_{j-1} + P_j}\right)\right]^{-1},$$

then $\mathbb{E}\left\{\Pi^{ND}(I_{j-1})\right\} < \mathbb{E}\left\{\Pi^{D}(I_{j-1})\right\}$ *for* $I_{j-1} > I_{\bar{m}}$.

[17] Lemma 1 is a general result for a profit function affected by the exchange rate, and can be used to analyze a setup in which the production costs of some firms are denominated in dollars. Intuitively, firms would set their price in dollars if they have costs in dollars and relatively more inelastic demands.

The income threshold in Proposition 4 describes two zones for the shopping list for any good j that depends on the expenditure in $j-1$ goods: zone I ($I_{j-1} < I_{\bar{m}}$) and zone II ($I_{j-1} > I_{\bar{m}}$).

Firms in zone I produce goods that are demanded by lower and higher income individuals. As the high-income individuals always consume the good independently of the currency in which the price is set, firms consider only what happens with the demand of the low-income individuals. As the low-income individuals have their assets in pesos, then fluctuations in the exchange rate do not affect their income and pricing in dollars would give uncertainty to the demand. Therefore, for goods in zone I, $\Pi_{12} = 0$ and the optimal pricing solution is in pesos.

On the other hand, goods in zone II are only consumed by individuals rich enough to dollarize their assets. We have that a depreciation of the currency increases both the price and the income of the individuals that demand goods in this zone. Given non-homothetic preferences we have that $\Pi_{12} > 0$, and so the depreciation reduces the elasticity of demand of these goods.

Recall that γ is a local measure of the concavity of the income distribution function related to income inequality such that a distribution with higher $\gamma(z)$ for every z exhibits a higher inequality. The second order condition for profit maximization implies that $\gamma(z) < 2$, establishing a limit to the concavity of the income distribution in order to have bounded profits. But it is not enough to satisfy this condition for dollarization. A sufficient condition to have the firms set prices in dollars is that $\gamma((I_{j-1} + P_j)/M) < \bar{\gamma}$ where $1 \leq \bar{\gamma} \leq 2$. This is the case when the concavity on the income distribution (income inequality) is moderate. When concavity (income inequality) is high, firms in zone II also prefer to set their price in pesos, because the demand is relatively inelastic to movements of the exchange rate. In this case, setting the price in dollars would increase the volatility of the profits, because when the price is in dollars the reduction in the volatility of the demand is low in comparison to the increase in volatility in the prices.

Corollary 1. *The number of goods priced in pesos, m, is defined by $I_{m-1} \leq I_{\bar{m}} < I_m$. Goods with index $j \leq m$ are priced in pesos, and those with $j > m$ are priced in dollars.*

Proposition 4 and Corollary 1 establish the main result of this chapter. The income threshold $I_{\bar{m}}$ fully characterizes the dollarization decision for firms.

Given moderate income inequality, for high enough index j, firms face demand composed only of customers that have dollarized their assets, thus they always find it optimal to set prices in foreign currency. The intuition of this result is that fluctuations in the exchange rate do not affect the elasticity of demand of these goods when the price is in dollars, but they do when the price is in pesos. Therefore, pricing in dollars is optimal because it reduces the uncertainty in the demand of these goods. As the index of the good j becomes lower, the participation of customers with non-dollarized assets increases, making demand for good j more elastic to pricing in dollars. For a small enough j, firms find it optimal not to set prices in dollars.[18]

Proposition 5. $I_{\bar{m}}$ *(the threshold of expenditure in goods priced in pesos) is an increasing and convex function of* θ_n *(the threshold of individuals with assets in pesos), i.e.* $\partial I_{\bar{m}}/\partial\theta_n > 0$ *and* $\partial^2 I_{\bar{m}}/\partial\theta_n^2 > 0$ *, for income distributions showing some degree of inequality.*

This proposition formally establishes the relationship between the threshold of individuals with assets in pesos, θ_n, and the threshold of expenditure in goods priced in pesos, $I_{\bar{m}}$. It shows that there is a causality relationship from θ_n to $I_{\bar{m}}$. When no individual finds it profitable to dollarize assets, then no firm finds it optimal to dollarize its price, because it would increase the uncertainty in the demand given that the elasticity of demand is higher when the price is in dollars. On the other hand, when all individuals dollarize their incomes, the income of all individuals changes proportionally to the depreciation rate, thus the demand faced by each firm becomes less elastic to the pricing in dollars, making demand more stable. When dollarization of assets is partial only some firms, those with sales concentrated on high-income customers with dollarized assets, set prices in dollars. The shape of the relationship between θ_n and $I_{\bar{m}}$ depends on the form of the income distribution.

9.3.3 Equilibrium with dollarization

When we introduce a second currency in the model, the equilibrium in this economy is defined as the number of firms and goods J, the number of firms that set the price

[18] Note that with some firms facing costs in dollars and homothetic preferences, the price setting decision is similar to the invoicing decision faced by an exporting firm featuring decreasing returns to scale, as in Bacchetta and van Wincoop (2005). In this case, firms set prices in dollars when the price elasticity is low, the higher their market share and the more differentiated their goods. With non-homothetic preferences, this result holds also for a constant returns to scale technology.

in pesos m, the mass of individuals that maintain their assets in pesos n, the set of consumption bundles $\{x(\theta)_j\}$ for $j = 0, 1, \ldots, J$ and $\theta \in [\underline{\theta}, \bar{\theta}]$, and the set of prices R, $\{P_j\}$ for $j = 0, 1, \ldots, m$, and $\{P_j^*\}$ for $j = m+1, \ldots, J$, such that all the individuals maximize expected utility subject to their budget constraints, all the firms maximize expected profits and the goods, factors and money market clear.

Given the fixed cost for asset dollarization c, the income distribution $F(\theta)$ and the exchange rate distribution $H(s)$, the Nash equilibrium that determines the levels of dollarization is given by the intersection of the schedules

$$\theta_n = (c/y)s^e/(1+s^e) \qquad \text{and} \qquad I_{\bar{m}} = M(\theta_n - G(\theta_n)) .$$

This intersection matches $n^* = F(\theta_n)$, the mass of individuals with assets in pesos, with $m^*(n^*)$, the number of goods priced in pesos. Therefore, the mass $1 - n^*$ of individuals, those with cash holdings $M_i > \theta_n M$, and the firms selling a good with index $j > m^*$ find it optimal to dollarize.

As shown in Figure 9.3, an increase in the cost c from c_0 to c_1 shifts up the θ_n schedule from $\theta_n(c_0)$ to $\theta_n(c_1)$, which in turn increases $I_{\bar{m}}$, from $I_m(c_0)$ to $I_m(c_1)$. Therefore, the increase in c reduces the mass of individuals, as well as the number of dollarized goods.[19]

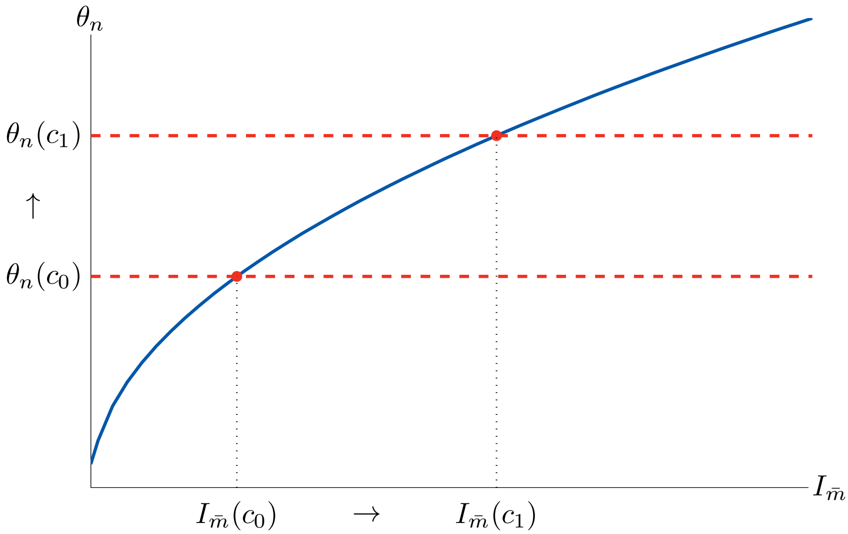
Similarly, Figure 9.4 shows that an increase in the expected depreciation rate, from s_0^e to s_1^e, shifts down the θ_n schedule and reduces the equilibrium values from $\theta_n(s_0^e)$ to $\theta_n(s_1^e)$, and from $I_{\bar{m}}(s_0^e)$ to $I_{\bar{m}}(s_1^e)$, thus increasing the dollarization for individuals and firms. It is important to note that the expected exchange rate only affects $I_{\bar{m}}$ through the effect on θ_n. The $I_{\bar{m}}$ schedule does not shift because for the firms, the only relevant moment of the distribution of s is the variance. Furthermore, unreported numerical simulations show that a higher variance of s shifts the $I_{\bar{m}}$ schedule to the left, increasing m for a given value of s^e.

9.4 Links between asset and transaction dollarization

The equilibrium under dollarization establishes a link between the mass of individuals with assets in dollars and the number of firms that set their prices in dollars. Aggregating individual decisions of firms and individuals, we can establish a link between asset and transaction dollarization. We first define both concepts.

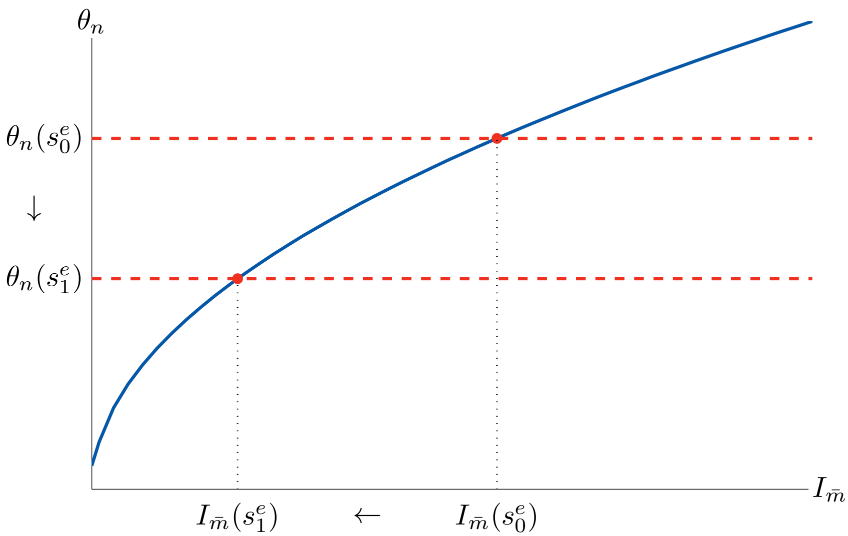
[19] Proposition 5 states that $I_{\bar{m}}$ is a convex function in θ_n. However, note that as we plot in Figures 9.3 and 9.4 the inverse function $\theta_n = I_{\bar{m}}^{-1}(\cdot)$, the curve is concave.

Figure 9.3 *Comparative statics: An increase in c*



Source: Authors' own elaboration.

Figure 9.4 *Comparative statics: An increase in s^e*



Source: Authors' own elaboration.

Definition 1 (Asset dollarization, AD). *The ratio of the sum of income of those individuals who dollarize to the sum of income of the total population:*

$$\mathrm{AD} = \frac{\displaystyle\int_{\theta_n}^{\bar{\theta}} PY(\theta)dF(\theta)}{\displaystyle\int_{\underline{\theta}}^{\bar{\theta}} PY(\theta)dF(\theta)}.$$

Asset dollarization is a decreasing function of n: the higher the mass of individuals who have chosen not to dollarize, the lower the ratio of asset dollarization in the economy. Note that we are using the income distribution ex-ante the depreciation of the currency, in order to control for income effects that may follow a depreciation.

It is important to mention that our measure of asset dollarization is directly comparable with measures of dollarization associated with the financial system only under the assumption that the cost of participating in the financial system is the same as the cost of participating in the exchange market. In countries with a history of dollarization, the cost of participating in the exchange market is usually much lower than the cost of participating in the financial system. Therefore, in those cases, our measure of dollarization will be systematically higher than those associated to the financial system.

Definition 2 (Transaction dollarization, TD). *The ratio of the sum of sales of firms with prices in dollars to the sum of sales over the whole spectrum of goods.*

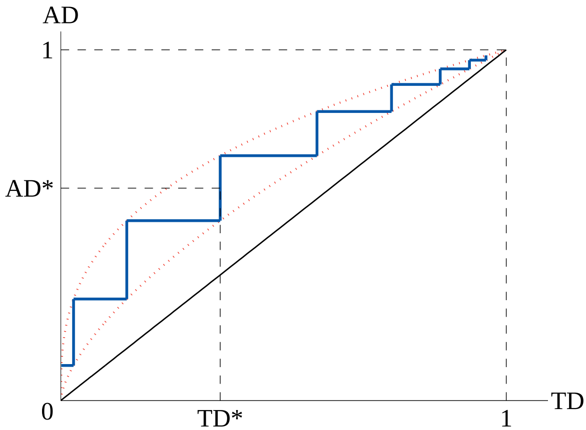
$$\mathrm{TD} = \frac{\sum_{j=m+1}^{J} P_j^* x_j^d}{\sum_{j=1}^{m} P_j x_j^d + \sum_{j=m+1}^{J} P_j^* x_j^d}.$$

Transaction dollarization is a strictly decreasing function of m, the number of goods that set prices in domestic currency. Also AD is a decreasing function of n, the mass of individuals who dollarize their assets.

Proposition 5 states that m is an increasing function of n. Thus, we can establish that TD is also an increasing function of AD and that AD $\geq$ TD. Proposition 6 follows.

Proposition 6. AD *is higher than* TD *for* AD $\in (0,1)$*, and* AD = TD *for* AD = $\{0,1\}$.

Figure 9.5 *Asset dollarization and transaction dollarization*



Source: Authors' own elaboration.

AD is higher than TD because individuals with assets in dollars consume both goods in pesos and in dollars, thus the proportion of goods traded in dollars would be smaller than the proportion of assets held in dollars. This is true even when some individuals with assets in pesos consume goods in dollars, because those goods account for a small fraction of their consumption basket. The exact shape of the relationship between AD and TD depends on the income distribution; more precisely, when the proportion of total income in the hands of the higher income individuals is higher, the difference between AD and TD would be higher. Note that, as shown in Figure 9.5, because of the discrete number of goods, TD is a step function of AD, and as the number of goods become larger, the steps in the function become smaller.

In our model, AD causes TD but not the other way around: when no individual finds it optimal to dollarize her assets, no firm has the incentives to set prices in dollars. Moreover, AD is independent of TD because of the linearity of individuals' preferences in the number of goods consumed. With a more general preferences specification, the individual portfolio decision will depend on m, the number of goods with peso prices. Therefore, m and n will be determined simultaneously, but the equilibrium value for m, described in this chapter, is a lower bound for the dollarization decisions of firms in this more general case.

The model explains why even in economies with very high levels of inflation domestic currency remains in circulation, and it is not fully replaced by a foreign currency as a medium of exchange and unit of account. In the model, if there exists some degree of income inequality, the use of dollars is not an option for the segment of the population with lower income. Therefore, firms producing goods whose demand is concentrated in low-income customers will find it optimal to set prices in pesos. In a more egalitarian society, everything else being equal, the model predicts that both asset and price dollarization will be higher. The model also explains why in countries with remarkably high levels of asset dollarization (such as Argentina, Bolivia and Peru), the levels of transaction and price dollarization are relatively low. Moreover, our results suggest that policy makers that are looking for policies aimed at reducing dollarization should focus mainly on reducing asset dollarization, because price and transactions dollarization will endogenously follow the pattern of asset dollarization.

Even though it is beyond the scope of this chapter to explain why dollarization is persistent, the model sheds some light on this issue. Persistence in asset dollarization can be generated if the cost of participating in the exchange market falls in parallel with expected level of depreciation. If this cost is small enough even for very small levels of expected depreciation, the levels of asset dollarization may remain high.[20]

The model also has implications regarding the pass-through effect of a depreciation on prices. In the short-run, the pass-through is approximately equal to TD because transactions in dollars increase proportionally to the depreciation rate, i.e. $\dot{P} \simeq s\text{TD}$. However, one period later, the long-run in our model, the pass-through is equal to AD, i.e. $\dot{P}' = s\text{AD}$. This is so because the income gained from a depreciation is distributed across all the individuals in the next period through the mutual fund, therefore the general level of prices P increases proportionally. If asset dollarization were equal to 1, then the nominal increase in profits and transactions would be equal to the depreciation rate. For levels of AD less than 1, both the short-run and long-run pass-through would be lower than 1. Moreover, for a given c, a higher expected depreciation rate would imply a higher degree of pass-through in the short and long-run.

9.5 Closing remarks

In this chapter we have presented a simple model that explains the pattern of dollarization across types of goods, and provides a theoretical link between asset and transaction dollarization. The model shows that asset dollarization causes price

[20] See Uribe (1997) and Winkelried and Castillo (2010) for surveys and discussion on the persistence in transaction and asset dollarization, respectively.

dollarization and that income distribution plays an important role in explaining the pattern of price dollarization. In particular, we find that for income distributions that show some degree of inequality, necessity goods, those associated with the consumption of low-income customers, have prices in pesos, whereas luxury goods have prices in dollars. Furthermore, the model shows that asset dollarization is larger than transaction dollarization, and the gap is increasing in the degree of inequality.

Although the model is simple, it captures reasonably well the main stylized facts we intended to explain. However, we aim to explore some extensions to the model. In particular, we would like to generalize the preferences setting to introduce some degree of substitution among goods. Also, multiperiod decision-making is considered in our future research agenda.

9.A Appendix: Proofs

Here, we provide the proofs of the Propositions and the Lemma in the text. The proof of Proposition 2 can be found in the text.

9.A.1 Proposition 1

Recall that $z = I_j/M$. The second order profit maximization condition implies

$$\frac{\partial^2 \Pi_j}{\partial P_j^2} = -2F'(z) - \left(\frac{P_j - R/\lambda_k}{M}\right) F''(z) = F'(z)\left[\gamma(z) - 2\right] < 0\,,$$

where the second equality uses (9.14). Since $F(z)$ is increasing, it follows that $\gamma(z) < 2$.

On the other hand, the optimal price for good j satisfies $P_j/M = G(z)$. Given the definition of $G(z)$, it follows that $G'(z) = \gamma(z) - 1$. Thus, the price has a finite solution if $\gamma(z) < 2$ or $G'(z) < 1$. Moreover, prices would be locally decreasing [resp. increasing] if $G'(z) < 0$ [resp. $G'(z) > 0$], and this is given by $\gamma(z) < 1$ [resp. $\gamma(z) > 1$].

9.A.2 Proposition 3

First, the expected utility function can be written in a more convenient way. Consider that money holdings and the set of prices are functions of depreciation of the exchange rate s. Then, we can write $M(s_i) \in [\, M(\underline{s}), M(\bar{s})\,]$, $\{P_h(s)\}_{h=1}^{h=J}$ and the state-contingent utility function as:

$$U_i = h + \bar{\varepsilon}(\, M(s) - \bar{I}_h(s)\,) \qquad \text{for} \quad s_{h-1} < s < s_h\,.$$

This function indicates that if the realization of the exchange rate is $s \in (s_{h-1}, s_h)$, given a level of money holdings and a sequence of prices, the individual will be able to consume h goods, where s_h is given by $M(s_h) = I_{h-1}(s_h) + P_h(s_h)$. Using this state-contingent utility function, its expected value can be written as:

$$\begin{aligned}
\mathbb{E}\{U_i\} &= \sum_{h=2}^{J} \int_{s_{h-1}}^{s_h} \left[h + \bar{\varepsilon}\left(M(s) - I_h(s)\right)\right] dH(s) + \int_{\underline{s}}^{s_1} \bar{\varepsilon} M(s) dH(s) \\
&= \bar{\varepsilon} \int_{\underline{s}}^{\bar{s}} M(s) dH(s) + \sum_{h=2}^{J} \int_{s_{h-1}}^{s_h} \left[h + \bar{\varepsilon}\left(M(s) - I_h(s)\right)\right] dH(s) \\
&\equiv \bar{\varepsilon}\, \mathbb{E}\left\{M\right\} + \Psi(P_1, P_2, \ldots, p_J, H)\,,
\end{aligned}$$

where $\Psi(P_1, P_2, \ldots, p_J, H)$ is the second summation in the second equality.

Recall that the level of cash holdings with dollarization is given by $(M_i - Pc)(1+s)$ and without dollarization by M_i. Thus, the corresponding expected utility functions with and without dollarization are given by:

$$\begin{aligned}
\mathbb{E}\left\{U_i^{ND}\right\} &= \bar{\varepsilon} M_i + \Psi^{ND}(P_1, P_2, \ldots, p_J, H)\,, \\
\mathbb{E}\left\{U_i^{D}\right\} &= \bar{\varepsilon}(M_i - Pc)(1 + s^e) + \Psi^{D}(P_1, P_2, \ldots, p_J, H)\,.
\end{aligned}$$

For levels of risk such that $\sigma^2 \to 0$ we obtain $\Psi^{ND} = \Psi^{D}$. Then, an individual will find it optimal to dollarize their income when $\mathbb{E}\{U_i^D\} > \mathbb{E}\{U_i^{ND}\}$, an inequality that holds if $\bar{\varepsilon}(M_i - Pc)(1 + s^e) > \bar{\varepsilon} M_i$. The result in Proposition 3 follows.

9.A.3 Lemma 1

A second order Taylor expansion of $\Delta(s, P^*, P) \equiv \Pi(P^*(1+s), s) - \Pi(P, s)$ around the expected depreciation gives:

$$\begin{aligned}
\Delta(s, P^*, P) = \Delta(s^e, P^*, P) + (s - s^e) \left.\frac{\partial \Delta(s, P^*, P)}{\partial s}\right|_{s=s^e} + \cdots \\
\cdots + \frac{(s - s^e)^2}{2} \left.\frac{\partial^2 \Delta(s^e, P^*, P)}{\partial s^2}\right|_{s=s^e} + O\left(\left\|s^3\right\|\right)\,.
\end{aligned}$$

For this expansion to hold, we require the profit function to be continuous and twice differentiable on s. Upon taking expectations:

$$\mathbb{E}\{\Delta(s,P^*,P)\} = \Delta(s^e,P^*,P) + \frac{\sigma^2}{2}\left.\frac{\partial^2\Delta(s^e,P^*,P)}{\partial s^2}\right|_{s=s^e} + O\left(\left\|s^3\right\|\right) .$$

Thus:

$$\frac{\partial\mathbb{E}\{\Delta(s,P^*,P)\}}{\partial\sigma^2} = \frac{1}{2}\left.\frac{\partial^2\Delta(s,P^*,P)}{\partial s^2}\right|_{s=s^e} .$$

Note that:

$$\begin{aligned}\frac{\partial^2\Delta(s,P^*,P)}{\partial s^2} &= \frac{\partial}{\partial s}\left[P^*\Pi_1(P^*(1+s),s) + \Pi_2(P^*(1+s),s) - \Pi_2(P,s)\right]\\ &= (P^*)^2\Pi_{11}(P^*(1+s),s) + P^*\Pi_{12}(P^*(1+s),s) + P^*\Pi_{21}(P^*(1+s),s) + \cdots\\ &\qquad \cdots + \Pi_{22}(P^*(1+s),s) - \Pi_{22}(P,s) .\end{aligned}$$

Therefore:

$$\begin{aligned}\left.\frac{\partial^2\Delta(s,P^*,P)}{\partial s^2}\right|_{s=s^e} &= (P^*)^2\Pi_{11}(P^*(1+s^e),s^e) + 2P^*\Pi_{12}(P^*(1+s^e),s^e) + \cdots\\ &\qquad \cdots + \Pi_{22}(P^*(1+s^e),s^e) - \Pi_{22}(P,s^e) .\end{aligned}$$

Let $\Pi_{11} \equiv \Pi_{11}(P,s^e)$, $\Pi_{22} \equiv \Pi_{22}(P,s^e)$ and $\Pi_{12} \equiv \Pi_{12}(P,s^e)$. If the approximation is such that $\sigma^2 \to 0$, then $P = P^*(1+s^e)$ or $P^* = P/(1+s^e)$, and the above expression simplifies to

$$\left.\frac{\partial^2\Delta(s,P^*,P)}{\partial s^2}\right|_{s=s^e,\sigma^2\to 0} = \left(\frac{P}{1+s^e}\right)^2\Pi_{11} + 2\left(\frac{P}{1+s^e}\right)\Pi_{12} + \Pi_{22} - \Pi_{22} .$$

Note that the last two terms cancel out. The result from the Lemma follows upon replacing this finding into the equality involving $\partial\mathbb{E}\{\Delta(s,P^*,P)\}/\partial\sigma^2$ above.

9.A.4 Proposition 4

From Lemma 1 the condition for dollarization for the firms is:

$$2\Pi_{12}(P,s^e,I_{j-1}) + \frac{P}{1+s^e}\Pi_{11}(P,s^e,I_{j-1}) > 0 .$$

Let

$$z = \frac{I_{j-1} + P_j}{M} \qquad \text{and} \qquad w = \frac{1}{1+s^e}\frac{I_{j-1} + P_j}{M} + \frac{c}{y}\,.$$

Given the profit function in equation (9.25), the dollarization condition becomes:

$$\begin{aligned} -\frac{P_j}{1+s^e}\left[2F'(z) + P_j F''(z)\right] &< 0 \quad \text{if } I_{j-1} < I_{\bar{m}} \\ \left(\frac{1}{1+s^e}\right)^2\left(\frac{I_{j-1}+P_j}{M}\right)\left[2F'(w) + \left(2 - \frac{P_j}{I_{j-1}+P_j}\right)\frac{P_j/M}{1+s^e}F''(w)\right] &< 0 \qquad \text{otherwise} \end{aligned}$$

The dollarization condition for firms is never satisfied for $I_{j-1} < I_{\bar{m}}$, and it would be satisfied for $I_{j-1} > I_{\bar{m}}$ if:

$$\gamma(w) < \left[1 - \frac{1}{2}\frac{P_j}{I_{j-1}+P_j}\right]^{-1},$$

as stated in the text.

Moreover, from Proposition 1 we obtain $\gamma(w) < 2$, which is not enough to satisfy the condition for dollarization for the firms. A sufficient condition to have that the firms set prices in dollars is that $\gamma(w) < \bar{\gamma}$, where $1 \leq \bar{\gamma} \leq 2$.

9.A.5 Proposition 5

Given that $I_{\bar{m}} = M\left(\theta_n - G(\theta_n)\right)$, we obtain $\partial I_{\bar{m}}/\partial\theta_n = M(1 - G'(\theta_n)) > 0$ because $G^{'}(\theta_n) < 1$, as is satisfied by the second order conditions (Proposition 1).

The second order derivative is:

$$\frac{\partial^2 I_{\bar{m}}}{\partial\theta_n^2} = M\frac{\gamma(\theta_n)}{G(\theta_n)}\left[1 - 2\gamma(\theta_n) + \gamma(\theta_n)\frac{F'}{(F'')^2}F'''\right].$$

For a concave income distribution ($F'' < 0$), we have that this is positive

for any $\gamma(\theta_n) > 0$ if $F'\,F'''/(F'')^2 \geq 2$,

for $\gamma(\theta_n) \in \left(0, \frac{1}{2 - F'\,F'''/(F'')^2}\right)$ otherwise.

When $F''' > 0$ the density of the income distribution is convex, and this is the case for most income distributions. Additionally, when γ is small [resp. large], $F'\,F'''/(F'')^2$ tends to be higher [resp. lower] than 2. Thus, the condition for the convexity would not be satisfied for distributions with high values of γ. Such is the case, for instance, of the exponential distribution where $\gamma(z) = F'\,F'''/(F'')^2 = 1$ for every z.

9.A.6 Proposition 6

Consider AD as in Definition 1 and TD as in Definition 2, which can be written as:

$$\text{TD} = \frac{\sum_{j=m+1}^{J} P_j \left[1 - F\left(\frac{I_j}{M}\right)\right]}{\sum_{j=1}^{J} P_j \left[1 - F\left(\frac{I_j}{M}\right)\right]} .$$

Since total income is equal to total expenditure in goods and on leisure:

$$\int_{\underline{\theta}}^{\bar{\theta}} Y(\theta) dF(\theta) = \sum_{j=1}^{J} P_j \left[1 - F\left(\frac{I_j}{M}\right)\right] + P_0 \int_{\underline{\theta}}^{\bar{\theta}} x_0(\theta) dF(\theta) .$$

On the other hand:

$$\int_{\theta_n}^{\bar{\theta}} Y(\theta)\, dF(\theta) = \sum_{j=m+1}^{J} P_j \left[1 - F\left(\frac{I_j}{M}\right)\right] + I_m \left(1 - F(\theta_n)\right) + P_0 \int_{\theta_n}^{\bar{\theta}} x_0(\theta) dF(\theta) .$$

It follows that:

$$\text{AD} = \frac{\sum_{j=m+1}^{J} P_j \left[1 - F\left(\frac{I_j}{M}\right)\right] + I_m \left(1 - F(\theta_n)\right) + P_0 \int_{\theta_n}^{\bar{\theta}} x_0(\theta) dF(\theta)}{\sum_{j=1}^{J} P_j \left[1 - F\left(\frac{I_j}{M}\right)\right] + P_0 \int_{\underline{\theta}}^{\bar{\theta}} x_0(\theta) dF(\theta)}$$
$$> \frac{\sum_{j=m+1}^{J} P_j \left[1 - F\left(\frac{I_j}{M}\right)\right]}{\sum_{j=1}^{J} P_j \left[1 - F\left(\frac{I_j}{M}\right)\right]} \equiv \text{TD} ,$$

given that:

$$I_m \left(1 - F(\theta_n)\right) + P_0 \int_{\theta_n}^{\bar{\theta}} x_0(\theta) dF(\theta) > P_0 \int_{\underline{\theta}}^{\bar{\theta}} x_0(\theta_i) dF(\theta) ,$$

which, in turn, follows from:

$$I_m \left(1 - F(\theta_n)\right) > P_0 \left(1 - F(Y_n)\right) \max \left\{x_0(\theta)\right\} > P_0 \int_{\underline{\theta}}^{\bar{\theta}} x_0(\theta) dF(\theta) ,$$

for any $\theta_n \in (\underline{\theta}, \bar{\theta})$. From this, it is also possible to see that when $\theta_n = \underline{\theta}$, then AD = TD = 0, whereas when $\theta_n = \bar{\theta}$, then AD = TD = 1.

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