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DD/05/03

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DOCUMENTO DE DISCUSIÓN

DD/05/03

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DD/05/03

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Financial Dollarization and the Size of the Fear *

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Julio 2005

Abstract

Based on the significance of a Minimum Variance Portfolio (MVP) for the understanding of dollarization equilibria, a significant strand of the debate concerned with the driving forces behind this phenomenon has focused on analyzing the determinants of the relative volatility of inflation vis-à-vis real depreciation. This analysis contributes in the identification of those factors by extending the basic CAPM formulation via the introduction of credit risk that is directly linked to the shock that determines real returns for dollar denominated assets: unanticipated shifts in the real exchange rate. We show this ingredient can end up altering the perceived relative volatility of peso and dollar assets in a way that fuels financial dollarization (by increasing the relative hedging opportunities offered by the latter). We calibrate our model using Peruvian data for the period 1998-2004, and its predictions show a better fit with observed financial dollarization ratios than those of the basic CAPM model.

Key words: Financial dollarization, Minimum Variance Portfolio, Perú.

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* The authors would like to thank Carlos Gallardo for excellent research assistance. As usual, any remaining errors are ours.



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1 Motivation

While the term “dollarization” is broad enough to cover the process by which the national currency is substituted by a foreign one in any of its three functions, it must be stressed that the early literature on the topic² was particularly concerned with the role of money as a means of exchange and, thus, dollarization was regarded as a currency substitution phenomenon. Under this scenario, one could claim that dollarization complicates and could ultimately render monetary policy as ineffective due to the instability of money demand. Clearly, this argument proves particularly relevant when the phenomenon behind the term “dollarization” is currency substitution, and when monetary policy is implemented by targeting some narrow monetary aggregate.

Despite the presence of severe data restrictions when it comes to account for cash holdings in foreign currency, empirical evidence points towards a significant reduction of the currency substitution phenomenon in most emerging economies during the last decade. On the other hand, indicators which rely on less liquid assets (like the share of bank deposits in foreign currency in broad money) have shown a positive evolution³. In the light of this evidence, the center of attention in the dollarization literature has shifted from the early concept of currency substitution, to privilege the role of money as a store of value under the concept of “asset substitution”. By asset substitution we understand the process by which the local currency is rejected as a store of value leading to “financial dollarization”, namely, the holding by residents of foreign currency denominated assets and liabilities (Levy Yeyati (2003 and 2004)).

To the extent that our emphasis when talking about “dollarization” has changed, the reasons that justify our concerns regarding this phenomenon also require a revision. While empirical evidence regarding the consequences of financial dollarization on the effectiveness of monetary policy

² See Calvo and Vegh (1997) for a comprehensive survey on this topic.

³ See Reinhart, *et al.* (2003) for an excellent survey on the evolution of dollarization in the developing world. By means of a broad measure of dollarization, they conclude that both the degree and incidence of this phenomenon have increased significantly over the last two decades. On the other hand, and via an indirect measure based on the average velocity of base money, they find evidence in favor of the hypothesis that the fall in the demand for domestic currency for transactional purposes (due to the high inflation episode of the 1980s) has been abated by the late 1990s.

seems inconclusive⁴, stronger arguments (both theoretical and empirical) can be found relating dollarization to financial fragility. This fragility stems from a combination of large and frequent exchange rate shocks and the existence of currency mismatches somewhere in the economy. These mismatches are typically introduced in the balance sheet of “liability dollarized” firms working in the non-tradable sector. In analytical grounds, and if we link financial costs to entrepreneurial net worth in a manner similar to that suggested in the “financial accelerator” literature⁵, currency mismatches will lead, in the event of a real depreciation, to a deterioration in firm’s net worth and access to credit, and this will amplify the real effects of a negative external shock⁶.

Now that consensus has been reached regarding the reasons that justify our concern about the dollarization phenomenon, the understanding of its driving forces and the possibility of suggesting a dedollarization agenda have recently been placed at the forefront of the policy debate.

The literature emphasizes two approaches to understand the causes of financial dollarization. The first approach (stressed in Levy Yeyati (2003) and Broda and Levy Yeyati (2003)) underlines the role of the regulatory framework. If there is a relatively high coverage and no discrimination against dollar deposits under the deposit insurance scheme, the banking system will fail to internalize the exchange rate risk in their pricing decisions.

In an environment characterized by a high correlation between exchange rate risk and banks’ solvency (a salient feature of liability dollarized economies), banks will fail to internalize the higher cost of dollar deposits relative to local currency (peso) deposits in the event of a depreciation, since it is precisely under such event that the bank is more likely to default. This, combined with the higher peso-dollar spread demanded by risk neutral depositors under a

⁴ The analysis presented in Reinhart, *et al.* (2003) suggests that a high degree of dollarization does not preclude monetary policy from attaining its goal of price stability. Levy Yeyati (2004), on the other hand, finds a positive relation between financial dollarization and inflation rates.

⁵ See Bernanke and Gertler (1989 and 1999).

⁶ General equilibrium models which address the consequences of currency mismatches due to liability dollarization within a financial accelerator framework can be found in Céspedes, *et al.* (2000), Gertler, *et al.* (2001) and Castro, *et al.* (2004).

liquidation policy that recognizes the denomination of claims⁷, makes dollar funding more attractive for banks and induces deposit dollarization. This effect is reinforced by the existence of a deposit insurance scheme that fails to discriminate between currencies. In fact, the deposit insurance, by increasing the fraction of dollar deposits recovered, widens the peso-dollar spread demanded by depositors, while banks still fail to internalize the higher costs of dollar funding in the bad states of nature, at the expense of the deposit insurance agency.

Besides the explicit coverage provided by the deposit insurance fund, the scenario described above could also be the result of an implicit insurance provided by government bail-outs implemented to avoid systemic risk. This argument (so-called *too-many-to-fail*, (Levy Yeyati (2003)) relies, precisely, in the assumption that the government will have to intervene ex post (through debt buyout programs or capital strengthening programs) to avoid a financial crisis.

Moreover, and at the other side of the balance sheet, the existence of an explicit or implicit insurance will also create an incentive for financial intermediaries to avoid transferring all the exchange rate risk to creditors when funding peso-generating projects. This will broaden currency mismatches (in banks' or firms' balance sheets) and increase the correlation between exchange rate risk and default risk. As a consequence, the driving force behind Broda and Levy Yeyati's (2003) model will be reinforced leading to more financial dollarization: dollar assets will become even more attractive for depositors, while banks' will now have weaker incentives to internalize the costs of dollar funding.

Moving towards risk averse depositors implies moving towards the second argument when talking about the driving forces behind dollarization: the portfolio approach. This approach stresses the importance of the relative volatility between inflation and real depreciation as a key determinant of financial dollarization. Ize and Levy Yeyati (2003) use an asset substitution model, CAPM (Capital Assets Portfolio Model), to formalize the previous statement. In particular, their model predicts that the degree of deposit and credit dollarization (given by the equilibrium in the financial market) is determined by the portfolio that ensures a

⁷ Dollar claims under a liquidation are recognized at the expense of peso claims when the liquidation has been triggered by a depreciation.

minimum variance. This portfolio is a function of inflation and real depreciation volatilities. Thus, the minimum variance portfolio (MVP) is the natural benchmark to measure the degree of financial dollarization, and relate it to macroeconomic variables which might be influenced by policy decisions.

Given the importance of second moments in the composition of depositors and creditors portfolio, an increase in the relative volatility of inflation (with respect to real depreciation volatility) will increase the dollarization ratio. This happens as the relative hedging opportunities offered by domestic currency assets fall. Based on the importance of relative volatilities, this approach should allow us to explain high and persistent financial dollarization ratios despite the introduction of a successful stabilization program. In fact, the MVP should not change if this program is accompanied by a policy aimed at mitigating exchange rate shocks.

If we confront the above argument with the data, however, we can identify significant and systematic deviations between basic CAPM predictions and observed dollarization ratios. Peru is a salient case. In particular, the relative volatility of inflation vis-à-vis real depreciation has experienced a sharp decline in the last decade, while financial dollarization has remained around 70%. In fact, the dollar composition of the Peruvian MVP according to Ize and Levy Yeyati's (2003) setting systematically underestimates observed financial dollarization.

As in the current literature (see, for example, Ize (2005)), we do agree in the relevance of a MVP as an appropriate tool to understand, measure and ultimately design policy options aimed at mitigating financial dollarization. However, and confronted with the empirical evidence, we also believe there are several elements missing in the basic CAPM formulation for it to yield the MVP around which observed dollarization should lie.

In fact, and if this model's outcomes rely on the hedging opportunities offered by peso and dollar denominated assets (via their perceived relative volatility), the existence of a deposit insurance should also have a role in the portfolio approach. Moreover, and as already mentioned, another salient feature of financially dollarized economies is the existence of a

high correlation between exchange rate risk and default risk. Therefore, real exchange rate fluctuations should also be allowed to affect perceived relative volatilities through the existence of a default scenario for the local portfolio.

In this way, we will be able to build an argument that draws from the two approaches discussed above. In particular, portfolio considerations will provide the framework to model the relative attractiveness of dollar assets as a hedging instrument. On the other hand, the two basic ingredients of the regulatory framework approach (the existence of a default scenario linked to real exchange shocks and the presence of currency-blind regulations), will enrich the set of determinants of this “relative attractiveness” and provide new means to understand (and policy options to modify) the dollar composition of the MVP.

Taking the above in consideration, the objective of this paper is to provide an extension to the basic CAPM formulation by allowing the existence of a default scenario for a sufficiently large real depreciation, and the presence of a deposit insurance scheme which is triggered under this scenario. The paper is organized in the following way. In Section 2 we present the model and discuss its results analytically. In Section 3 we calibrate the model with Peruvian data and compare its predictions with those of the basic CAPM. Finally, in Section 4 we conclude and suggest some avenues for further research.

2 The Model

In this section we extend Ize and Levy Yeyati’s (2003) minimum variance portfolio model. As in the original setting, depositor’s preferences are set to maximize:

$$U = E(r) - c_D \text{Var}(r)/2 \quad (1.)$$

where r is the real average return of the portfolio and $c_D > 0$ captures the degree of risk aversion. The key element in the model are the assets available for the portfolio choice and the type of shock that affects the realized return of each asset.

Following the definitions of the original model, there are three assets available for the portfolio choice: local currency deposits in the domestic banking system (DH), foreign currency deposits in the domestic banking system (DF), and cross-border foreign currency deposits (DC). Realized real returns (discounted with the domestic price index) of these three assets are given by: r^H , r^F and r^C , respectively. In accordance with the price index used to build real returns, r^H is subject to inflation surprises (μ_π), while r^F and r^C are both subject to real depreciation surprises (μ_s). In addition, it is assumed that real returns for locally held assets are subject to a confiscation shock (μ_c), which is uncorrelated with inflation and real depreciation surprises.

In order to extend this model and incorporate a default scenario linked to real exchange rate shocks, we further assume the existence of a critical size for the real depreciation surprise (δ^*) that triggers default. In terms of the discussion that motivates this analysis, this means we are assuming that “balance sheet effects” matter and have pervasive effects for a sufficiently large real depreciation⁸. The existence of a default scenario brings the deposit insurance scheme into our analysis. In particular, we assume that this scheme recognizes a percentage (α) of the realized real return on locally held deposits.

Given all the above, real returns for the three types of asset can be expressed as:

$$r^F = \begin{cases} \bar{r}^F + (\mu_s | \mu_s < \delta^*) + \mu_c & \text{if } \mu_s < \delta^* \\ \alpha_F [\bar{r}^F + (\mu_s | \mu_s \geq \delta^*) + \mu_c] & \text{if } \mu_s \geq \delta^* \end{cases} \quad (2.)$$

$$r^H = \begin{cases} \bar{r}^H - (\mu_\pi | \mu_s < \delta^*) + \mu_c & \text{if } \mu_s < \delta^* \\ \alpha_H [\bar{r}^H - (\mu_\pi | \mu_s \geq \delta^*) + \mu_c] & \text{if } \mu_s \geq \delta^* \end{cases} \quad (3.)$$

⁸ An example of this was the combined impact of the Asian and Russian crisis on the Peruvian economy. As a consequence of the crisis, seven banks (out of 25) were intervened and closed. A detailed account of the financial crisis of those years can be found in Morón and Loo-Kung (2003).

$$r^C = \begin{cases} \bar{r}^C + (\mu_s | \mu_s < \delta^*) & \text{if } \mu_s < \delta^* \\ \bar{r}^C + (\mu_s | \mu_s \geq \delta^*) & \text{if } \mu_s \geq \delta^* \end{cases} \quad (4.)$$

where \bar{r} denotes real returns in the absence of surprises, and α_F, α_H denote the percentage covered by the deposit insurance for locally held dollar and peso assets, respectively.

The confiscation shock is assumed to be distributed with mean zero and variance σ_C^2 . Inflation and real depreciation surprises are assumed to be jointly normally distributed⁹:

$$\begin{bmatrix} \mu_s \\ \mu_\pi \end{bmatrix} \sim N \left[\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_s^2 & \sigma_{s\pi} \\ \sigma_{s\pi} & \sigma_\pi^2 \end{pmatrix} \right] \quad (5.)$$

If we let x_F, x_C define the dollar shares of the portfolio, its real return will be given by:

$$r = (1 - x_F - x_C)r^H + x_F r^F + x_C r^C \quad (6.)$$

and its first two moments are defined by:

$$E(r) = (1 - x_F - x_C)E(r^H) + x_F E(r^F) + x_C E(r^C) \quad (7.)$$

$$\begin{aligned} \text{Var}(r) &= (1 - x_F - x_C)^2 \text{Var}(r^H) + x_F^2 \text{Var}(r^F) + x_C^2 \text{Var}(r^C) + 2(1 - x_F - x_C)x_F \text{Cov}(r^H, r^F) + \dots \\ &\dots + 2(1 - x_F - x_C)x_C \text{Cov}(r^H, r^C) + 2x_F x_C \text{Cov}(r^F, r^C) \end{aligned} \quad (8.)$$

Thus, and for a given degree of risk aversion, the optimal composition of depositors' portfolio is given by the solution to the following optimization problem¹⁰:

⁹ Normality will ease the analytical tractability of our results.

¹⁰ The reader can check the analytical solution to this problem in the appendix of Ize and Levy Yeyati (2003).

$$\begin{aligned} & \text{Max}_{x^F, x^C} (E(r) - c_D \text{Var}(r)/2) \\ & \text{s.t. (7) and (8)} \end{aligned}$$

As is Ize and Levy Yeyati's (2003) formulation, however, we are ultimately interested in the peso and dollar share of the minimum variance portfolio. If we let λ^F, λ^C define the dollar shares of the MVP (local and cross-border, respectively), $\lambda^* = \lambda^F + \lambda^C$ should yield the degree of financial dollarization around which the observed ratio should lie (the degree of "underlying dollarization", as defined by the authors).

Thus, λ^F, λ^C can be obtained from the values of x_F, x_C that solve:

$$\text{Min}_{x^F, x^C} (\text{Var}(r)) \quad \text{s.t. (8)}$$

From (8.) is clear that the solution to the above problem requires solving for the unconditional second moments of r^H , r^F and r^C . Given the way in which we have defined these real returns, their unconditional variances and covariances will be a function of their variances and means conditioned over the real depreciation surprise. Since real returns have been defined for two different states of nature over the entire support of the distribution of real depreciation shocks, it is worth noticing that conditioning would be trivial under full coverage of the deposit insurance ($\alpha_H = \alpha_F = 1$). Cross-border assets, on the other hand, are not covered by the deposit insurance but neither are subject to the default scenario considered in this analysis. Thus, cross-border returns mimic a situation where full coverage is granted. This implies that their variance can be directly computed as: $\text{Var}(r^c) = \sigma_s^2$.

Let $\phi(\cdot), \Phi(\cdot)$ define the pdf and cdf of the normal distribution; $\gamma^* = \delta^*/\sigma_s$ denote the normalized critical depreciation size (the critical shock expressed as a number of standard deviations); and $\rho = \sigma_{s\pi}/\sigma_s\sigma_\pi$. By using the general formula for the variance decomposition

in a joint distribution: $\text{Var}(y) = \text{Var}_x [E[y|x]] + E_x [\text{Var}[y|x]]$, it can be shown (see Appendix A) that variances for locally held assets are given by:

$$\begin{aligned}
\text{Var}(r^F) &= \Phi(\gamma^*) \left\{ \sigma_s^2 \left[1 - \left(\frac{\phi(\gamma^*)}{\Phi(\gamma^*)} \right)^2 - \gamma^* \frac{\phi(\gamma^*)}{\Phi(\gamma^*)} \right] + \sigma_C^2 \right\} + \dots \\
&\dots + \alpha_F^2 (1 - \Phi(\gamma^*)) \left\{ \sigma_s^2 \left[1 - \left(\frac{\phi(\gamma^*)}{1 - \Phi(\gamma^*)} \right)^2 + \gamma^* \frac{\phi(\gamma^*)}{1 - \Phi(\gamma^*)} \right] + \sigma_C^2 \right\} + \dots \\
&\dots + \Phi(\gamma^*) \left\{ \bar{r}^F [(1 - \Phi(\gamma^*))(1 - \alpha_F)] - \sigma_s \phi(\gamma^*) \left[\frac{1}{\Phi(\gamma^*)} - (1 - \alpha_F) \right] \right\}^2 + \dots \\
&\dots + (1 - \Phi(\gamma^*)) \left\{ \bar{r}^F [\Phi(\gamma^*)(\alpha_F - 1)] + \sigma_s \phi(\gamma^*) \left[\frac{\alpha_F}{1 - \Phi(\gamma^*)} - (\alpha_F - 1) \right] \right\}^2
\end{aligned} \tag{9}$$

$$\begin{aligned}
\text{Var}(r^H) &= \Phi(\gamma^*) \left\{ \sigma_\pi^2 \left[1 - \left(\frac{\rho\phi(\gamma^*)}{\Phi(\gamma^*)} \right)^2 - \gamma^* \frac{\rho^2\phi(\gamma^*)}{\Phi(\gamma^*)} \right] + \sigma_C^2 \right\} + \dots \\
&\dots + \alpha_H^2 (1 - \Phi(\gamma^*)) \left\{ \sigma_\pi^2 \left[1 - \left(\frac{\rho\phi(\gamma^*)}{1 - \Phi(\gamma^*)} \right)^2 + \gamma^* \frac{\rho^2\phi(\gamma^*)}{1 - \Phi(\gamma^*)} \right] + \sigma_C^2 \right\} + \dots \\
&\dots + \Phi(\gamma^*) \left\{ \bar{r}^H [(1 - \Phi(\gamma^*))(1 - \alpha_H)] + \frac{\sigma_{s\pi}}{\sigma_s} \phi(\gamma^*) \left[\frac{1}{\Phi(\gamma^*)} - (1 - \alpha_H) \right] \right\}^2 + \dots \\
&\dots + (1 - \Phi(\gamma^*)) \left\{ \bar{r}^H [\Phi(\gamma^*)(\alpha_H - 1)] - \frac{\sigma_{s\pi}}{\sigma_s} \phi(\gamma^*) \left[\frac{\alpha_H}{1 - \Phi(\gamma^*)} - (\alpha_H - 1) \right] \right\}^2
\end{aligned} \tag{10}$$

By the same token, unconditional covariances between the three assets considered are given by:

$$\begin{aligned}
\text{Cov}(r^F, r^H) &= \Phi(\gamma^*) \left\{ \sigma_C^2 - \sigma_{s\pi} \left[1 - \left(\frac{\phi(\gamma^*)}{\Phi(\gamma^*)} \right)^2 - \gamma^* \frac{\phi(\gamma^*)}{\Phi(\gamma^*)} \right] \right\} + \alpha_F \alpha_H (1 - \Phi(\gamma^*)) \left\{ \sigma_C^2 - \sigma_{s\pi} \left[1 - \left(\frac{\phi(\gamma^*)}{1 - \Phi(\gamma^*)} \right)^2 + \gamma^* \frac{\phi(\gamma^*)}{1 - \Phi(\gamma^*)} \right] \right\} \\
&+ \Phi(\gamma^*) \left\{ \bar{r}^F [(1 - \Phi(\gamma^*))(1 - \alpha_F)] - \sigma_s \phi(\gamma^*) \left[\frac{1}{\Phi(\gamma^*)} - (1 - \alpha_F) \right] \right\} \left\{ \bar{r}^H [(1 - \Phi(\gamma^*))(1 - \alpha_H)] + \frac{\sigma_{s\pi}}{\sigma_s} \phi(\gamma^*) \left[\frac{1}{\Phi(\gamma^*)} - (1 - \alpha_H) \right] \right\} \\
&+ (1 - \Phi(\gamma^*)) \left\{ \bar{r}^F [\Phi(\gamma^*)(\alpha_F - 1)] + \sigma_s \phi(\gamma^*) \left[\frac{\alpha_F}{1 - \Phi(\gamma^*)} - (\alpha_F - 1) \right] \right\} \left\{ \bar{r}^H [\Phi(\gamma^*)(\alpha_H - 1)] - \frac{\sigma_{s\pi}}{\sigma_s} \phi(\gamma^*) \left[\frac{\alpha_H}{1 - \Phi(\gamma^*)} - (\alpha_H - 1) \right] \right\}
\end{aligned} \tag{11}$$

$$\begin{aligned}
\text{Cov}(r^H, r^C) &= \Phi(\gamma^*) \left\{ \sigma_{s\pi} \left[\left(\frac{\phi(\gamma^*)}{\Phi(\gamma^*)} \right)^2 + \gamma^* \frac{\phi(\gamma^*)}{\Phi(\gamma^*)} - 1 \right] + \alpha_H (1 - \Phi(\gamma^*)) \left\{ \sigma_{s\pi} \left[\left(\frac{\phi(\gamma^*)}{1 - \Phi(\gamma^*)} \right)^2 - \gamma^* \frac{\phi(\gamma^*)}{1 - \Phi(\gamma^*)} - 1 \right] \right\} \right. \\
&- \alpha_s \phi(\gamma^*) \left\{ \bar{r}^H [(1 - \Phi(\gamma^*))(1 - \alpha_H)] + \frac{\sigma_{s\pi}}{\sigma_s} \phi(\gamma^*) \left[\frac{1}{\Phi(\gamma^*)} - (1 - \alpha_H) \right] \right\} \\
&+ \alpha_s \phi(\gamma^*) \left\{ \bar{r}^H [\Phi(\gamma^*)(\alpha_H - 1)] - \frac{\sigma_{s\pi}}{\sigma_s} \phi(\gamma^*) \left[\frac{\alpha_H}{1 - \Phi(\gamma^*)} - (\alpha_H - 1) \right] \right\}
\end{aligned} \tag{12.}$$

$$\begin{aligned}
\text{Cov}(r^F, r^C) &= \Phi(\gamma^*) \left\{ \sigma_s^2 \left[1 - \left(\frac{\phi(\gamma^*)}{\Phi(\gamma^*)} \right)^2 - \gamma^* \frac{\phi(\gamma^*)}{\Phi(\gamma^*)} \right] + \alpha_F (1 - \Phi(\gamma^*)) \left\{ \sigma_s^2 \left[1 - \left(\frac{\phi(\gamma^*)}{1 - \Phi(\gamma^*)} \right)^2 + \gamma^* \frac{\phi(\gamma^*)}{1 - \Phi(\gamma^*)} \right] \right\} \right. \\
&- \alpha_s \phi(\gamma^*) \left\{ \bar{r}^F [(1 - \Phi(\gamma^*))(1 - \alpha_F)] - \alpha_s \phi(\gamma^*) \left[\frac{1}{\Phi(\gamma^*)} - (1 - \alpha_F) \right] \right\} \\
&+ \alpha_s \phi(\gamma^*) \left\{ \bar{r}^F [\Phi(\gamma^*)(\alpha_F - 1)] + \alpha_s \phi(\gamma^*) \left[\frac{\alpha_F}{1 - \Phi(\gamma^*)} - (\alpha_F - 1) \right] \right\}
\end{aligned} \tag{13.}$$

Clearly, and given the above expressions, the optimal dollar share of the MVP will be far more convoluted than the solution proposed in the basic model. In fact, Ize and Levy Yeyati (2003) show that λ^* can be expressed as a simple ratio of second moments of inflation and real depreciation surprises:

$$\lambda^* = \frac{\sigma_\pi^2 + \sigma_{s\pi}}{\sigma_s^2 + \sigma_\pi^2 + 2\sigma_{s\pi}}, \tag{14.}$$

from where is clear that the relative volatility of inflation vis-à-vis real depreciation is a key determinant of the degree of underlying dollarization.

However, it is worth noticing that Ize and Levy Yeyati's setting and results are both a special case of this model. In particular, and as already mentioned, we have defined two states of nature which include the entire support of the distribution of real depreciation surprises. Therefore, we can identify two situations in which our model would mimic the basic model's results: (i) when conditioning is not binding (the shock required to trigger default is

sufficiently large; $\gamma^* \rightarrow \infty$)¹¹; or (ii) when full coverage is granted by the deposit insurance scheme ($\alpha_H = \alpha_F = 1$).

Given the above, our model's optimal composition of the MVP will be different from that of the basic setting under a situation where the probability of default is non-trivial (balance sheet effects constitute a latent risk for financial stability), and the deposit insurance scheme grants less than perfect coverage. In order to explore this novel feature, two key elements remain to be uncovered: in what direction does our model's results differ from those of the basic CAPM formulation, and what is the role of the size of the critical depreciation that triggers default.

2.1 The MVP and the size of the fear

As in the basic model, and for a sufficiently low country risk, one of the key determinants of the dollar share of the MVP is the volatility of peso returns relative to that of locally held dollar returns: $\text{Var}(r^H)/\text{Var}(r^F)$. In fact, it is clear from expressions (9.) and (10.) that for a sufficiently large γ^* or full coverage (and if we let $\sigma_c^2 \rightarrow 0$), the above ratio converges to σ_π^2/σ_s^2 .

Therefore, and in order to try to uncover some of the main implications of our model, we will focus on the new expressions given for the variances of returns for locally held peso and dollar assets. For simplicity, and in order to exploit the existence of a default scenario linked to real depreciation disturbances, we will assume a situation where no coverage is granted ($\alpha_H = \alpha_F = 0$). Given this, and if we let $\sigma_c^2 \rightarrow 0$, expressions (9.) and (10.) simplify to:

$$\begin{aligned} \text{Var}(r^F) = \sigma_s^2 & \left\{ \Phi(\gamma^*) - \gamma^* \phi(\gamma^*) - \frac{\phi(\gamma^*)^2}{\Phi(\gamma^*)} \right\} + \dots \\ & \dots + (\bar{r}^F)^2 \Phi(\gamma^*)(1 - \Phi(\gamma^*)) + \sigma_s^2 \phi(\gamma^*)^2 \left[\frac{1 - \Phi(\gamma^*)}{\Phi(\gamma^*)} \right] - 2\phi(\gamma^*) \sigma_s \bar{r}^F (1 - \Phi(\gamma^*)) \end{aligned} \quad (15.)$$

¹¹ Since we are considering a normal distribution for real depreciation surprises, a threshold value of 3 would suffice to guarantee that truncation is not relevant.

$$\begin{aligned} \text{Var}(r^H) = & \sigma_\pi^2 \left\{ \Phi(\gamma^*) - \gamma^* \rho^2 \phi(\gamma^*) - \frac{\rho^2 \phi(\gamma^*)^2}{\Phi(\gamma^*)} \right\} + \dots \\ & \dots + (\bar{r}^H)^2 \Phi(\gamma^*) (1 - \Phi(\gamma^*)) + \sigma_\pi^2 \rho^2 \phi(\gamma^*)^2 \left[\frac{1 - \Phi(\gamma^*)}{\Phi(\gamma^*)} \right] + 2\rho\phi(\gamma^*)\sigma_\pi\bar{r}^H(1 - \Phi(\gamma^*)) \end{aligned} \quad (16.)$$

Where the first expression at the right hand side of both equations represents the mean of the conditional variance, while the second and third sum up the variance of the conditional mean.

Let us now focus on the role of the size of the critical depreciation that triggers default. A smaller value for γ^* implies a higher probability of default and, in accordance with the title that motivates this analysis, let us portray small values of γ^* as implying the existence of a larger “fear”.

From (15.) and (16.) it is clear that a larger “fear” translates into a reduction in the mean of the conditional variance. In fact, the mean of the conditional variance is monotonically increasing in γ^* :

$$\begin{aligned} \lim_{\gamma^* \rightarrow \infty} \left\{ E \left[\text{Var} \left(r^F | \mu_s \right) \right] \Big|_{\alpha_F=0} \right\} = \sigma_s^2; \quad \lim_{\gamma^* \rightarrow -\infty} \left\{ E \left[\text{Var} \left(r^F | \mu_s \right) \right] \Big|_{\alpha_F=0} \right\} = 0 \\ \lim_{\gamma^* \rightarrow \infty} \left\{ E \left[\text{Var} \left(r^H | \mu_s \right) \right] \Big|_{\alpha_H=0} \right\} = \sigma_\pi^2; \quad \lim_{\gamma^* \rightarrow -\infty} \left\{ E \left[\text{Var} \left(r^H | \mu_s \right) \right] \Big|_{\alpha_H=0} \right\} = 0 \end{aligned} \quad (17.)$$

The variance of the conditional mean, on the other hand, is a non-monotonic function of γ^* . In particular, it will converge to zero as $\gamma^* \rightarrow \infty$ or $\gamma^* \rightarrow -\infty$. For sufficiently low absolute values of \bar{r}^H and \bar{r}^F (peso and dollar real returns in the absence of surprises), however, the first effect will dominate and we can claim that unconditional variances will fall as γ^* drops.

In other words, and in a situation where the effect over the conditional variance dominates, a larger “fear” will conduce to a reduction in the variance of both locally held peso and dollar assets. This occurs because we are effectively truncating the distribution from where real depreciation and inflation disturbances are drawn.

Now, and in order to translate these effects into the composition of the MVP, we need to analyze their implications on the relative variance of peso and dollar assets. For this, we will continue focusing on a situation where the effect of γ^* on the conditional variance dominates.

Thus, and for sufficiently low values of \bar{r}^H and \bar{r}^F , we have:

$$\frac{\text{Var}(r^H)}{\text{Var}(r^F)} \approx \frac{\sigma_\pi^2 \left\{ \Phi(\gamma^*) - \rho^2 \left[\gamma^* \phi(\gamma^*) + \frac{\phi(\gamma^*)^2}{\Phi(\gamma^*)} \right] \right\}}{\sigma_s^2 \left\{ \Phi(\gamma^*) - \left[\gamma^* \phi(\gamma^*) + \frac{\phi(\gamma^*)^2}{\Phi(\gamma^*)} \right] \right\}} \quad (18.)$$

$$\frac{\text{Var}(r^H)}{\text{Var}(r^F)} \approx \frac{\sigma_\pi^2}{\sigma_s^2} \frac{A}{B}; \quad A > B \text{ if } 0 \leq \rho < 1 \quad (19.)$$

Equations (18.) and (19.) suggest a corrected version of the relative variance of inflation and real depreciation (the key determinant of the composition of the MVP). In particular, and to the extent in which inflation and real depreciation surprises remain less than perfectly correlated, the correction term introduced (A/B) will be greater than one, and our model will be able to predict a larger dollar share in the MVP than the basic CAPM formulation.

Moreover, this correction term is a decreasing function of γ^* and, thus, the difference between this model's λ^* and Ize and Levy Yeyati's λ^* will be greater, the larger the size of the "fear".

$$\text{sign} \left[\frac{\partial A/B}{\partial \gamma^*} \right] = \text{sign} \left\{ (B - A) \phi(\gamma^*) + (B\rho^2 - A) \left[(\gamma^*)^2 \phi(\gamma^*) + \frac{\phi(\gamma^*)^2}{\Phi(\gamma^*)} \left(2\gamma^* + \frac{\phi(\gamma^*)}{\Phi(\gamma^*)} \right) \right] \right\} = (-)$$

since $A > B$

Summarizing, our model can explain deviations from the dollar share of the MVP predicted by the basic CAPM setting, which depend on the probability that a real depreciation surprise triggers a financial crisis and, thus, default for locally held assets (the size of the "fear"). In

particular, the optimal composition of our MVP can call for a larger share of dollar assets and, under this scenario, the degree of underlying dollarization will be an increasing function of the degree of credit risk.

The key element for this result is the link between credit risk and the shock that affects real returns for dollar assets: real exchange rate disturbances. This, together with the presence of inflation surprises that remain less than perfectly correlated with real depreciation shocks, implies that returns for dollar assets become relatively less volatile than peso returns.

In terms of the distributions from where shocks that affect dollar and peso returns are drawn, our claim is that if default can be driven by real depreciation surprises, truncation will directly act upon the distribution of dollar returns while only indirectly upon the distribution of peso returns¹², thus reducing the volatility of the former vis-à-vis that of the latter.

3 The Peruvian Fear

In the preceding section we have tried to uncover some of the main implications of our model, and analyze in which way can it predict a different composition of the MVP connecting this result to the existence of a non-trivial probability of default linked to real exchange rate disturbances.

In this section, our goal is to test this model and its implications using Peruvian data. For this, we simulated numerical solutions to the problem:

$$\begin{aligned} & \text{Min}_{x^F, x^C}(\text{Var}(r)) \\ & \text{s.t. } \text{Var}(r) = (1 - x_F - x_C)^2 \text{Var}(r^H) + x_F^2 \text{Var}(r^F) + x_C^2 \text{Var}(r^C) + 2(1 - x_F - x_C)x_F \text{Cov}(r^H, r^F) + \dots \\ & \dots + 2(1 - x_F - x_C)x_C \text{Cov}(r^H, r^C) + 2x_F x_C \text{Cov}(r^F, r^C) \\ & \text{and } 0 \leq x_F \leq 1, 0 \leq x_C \leq 1 \end{aligned}$$

¹² In the extreme case where $\rho = 0$, incidental truncation will be irrelevant for inflation shocks. This will imply that realizations from the entire support of the inflation distribution can occur, regardless of the realized real depreciation shock.

using expressions (9.) to (13.) and $\text{Var}(r^c) = \sigma_s^2$ to build all necessary second moments. Simulations were made for every year in the period 1998-2004, using observed inflation, real depreciation and average real returns to build $\sigma_\pi^2, \sigma_s^2, \sigma_{s\pi}, \bar{r}^H$, and \bar{r}^F . Average coverage from the deposit insurance scheme, on the other hand, was calculated as the weighted average of the coverage granted for each deposit size in the ranges reported by the National Banking Superintendency (NBS)¹³. Weights were the total value of deposits in each range, as reported by the NBS. For each year, we computed solutions for values of γ^* between 0.75 and 1.5.

Table and Graph No. 1 present the results of our simulation exercise and compare them with the predictions of the basic model. The first result that is worth noticing is that the dollar share of the basic model's MVP (as expressed in equation (14.)) systematically underestimates observed end-of-period dollarization ratios. In fact, the relative volatility of inflation with respect to that of the real exchange rate declined at a faster rate than financial dollarization during the 90s, opening a gap between observed and predicted dollarization.

Table No. 1
The Basic vs. Our Model's Predictions

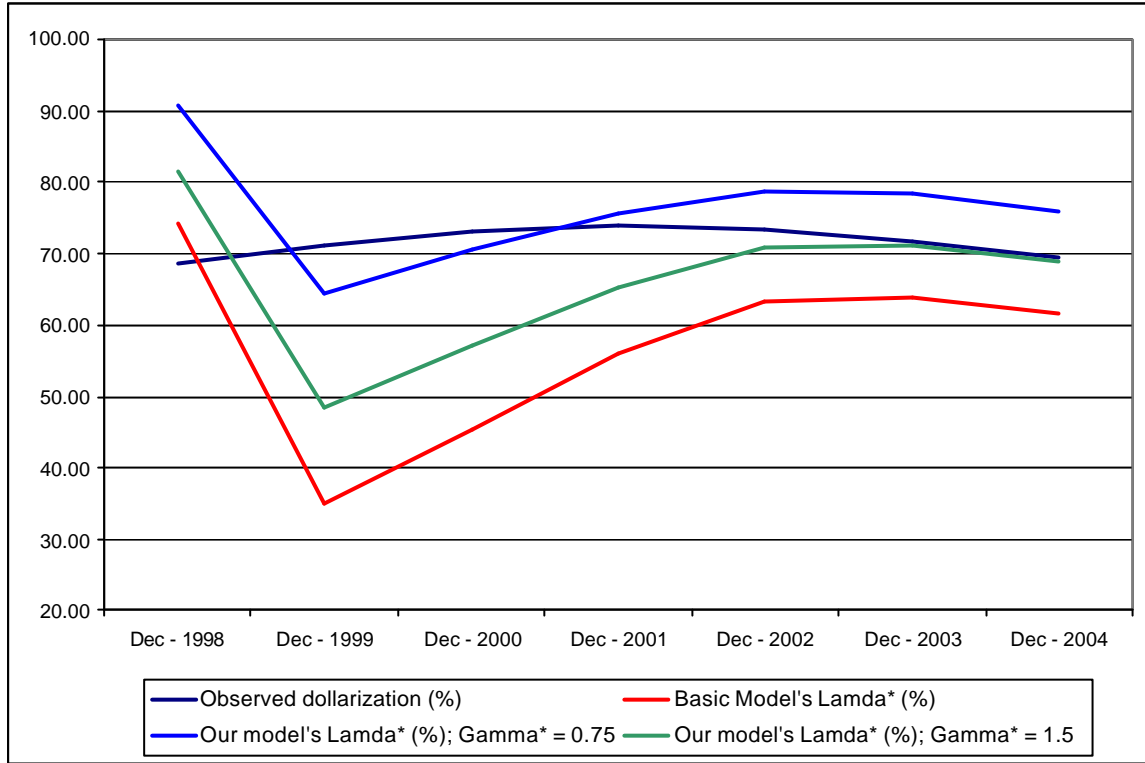
Period /1	Observed dollarization (%) /2	Basic model's l * (%)	Our model's l * g* = 0.75 (%)	Our model's l * g* = 1.5 (%)
Dec-1998	68.56	74.28	90.74	81.61
Dec-1999	71.11	34.90	64.39	48.30
Dec-2000	73.27	45.40	70.49	57.06
Dec-2001	73.90	55.92	75.43	65.26
Dec-2002	73.44	63.19	78.85	70.87
Dec-2003	71.63	63.69	78.44	71.01
Dec-2004	69.38	61.54	75.96	68.81

/1 Unconditional second moments for inflation and the real exchange rate, as well as average real returns for locally held dollar and peso (sol) assets, were computed using historic data with an increasing window size of 5 (for Dec-1998) to 11 (for Dec-2004) years. This allowed us to use all available information since Jan-1994.

/2 Computed as the ratio $(DF+DC) / (DF+DC+DH)$; DH expressed in dollars.

¹³ The Peruvian deposit insurance scheme recognized a maximum deposit value of around US\$ 22,000 during the period 1998-2004.

Graph No. 1
The Basic vs. Our Model's Predictions



The above result implies that the ratio $\sigma_{\pi}^2/\sigma_s^2$ is not a sufficient statistic to explain the dynamics of financial dollarization in the Peruvian economy. Recall that one of the main conclusions derived from the basic model is that we should be able to explain high and persistent financial dollarization despite the introduction of a successful stabilization program, if this is accompanied by a fear-of-floating-type behavior by the central bank. Obviously, this argument (summarized in the ratio $\sigma_{\pi}^2/\sigma_s^2$) can help explain part of the phenomenon's persistence, but our claim is that there is still a missing driving force behind financial dollarization which our model appears to be considering.

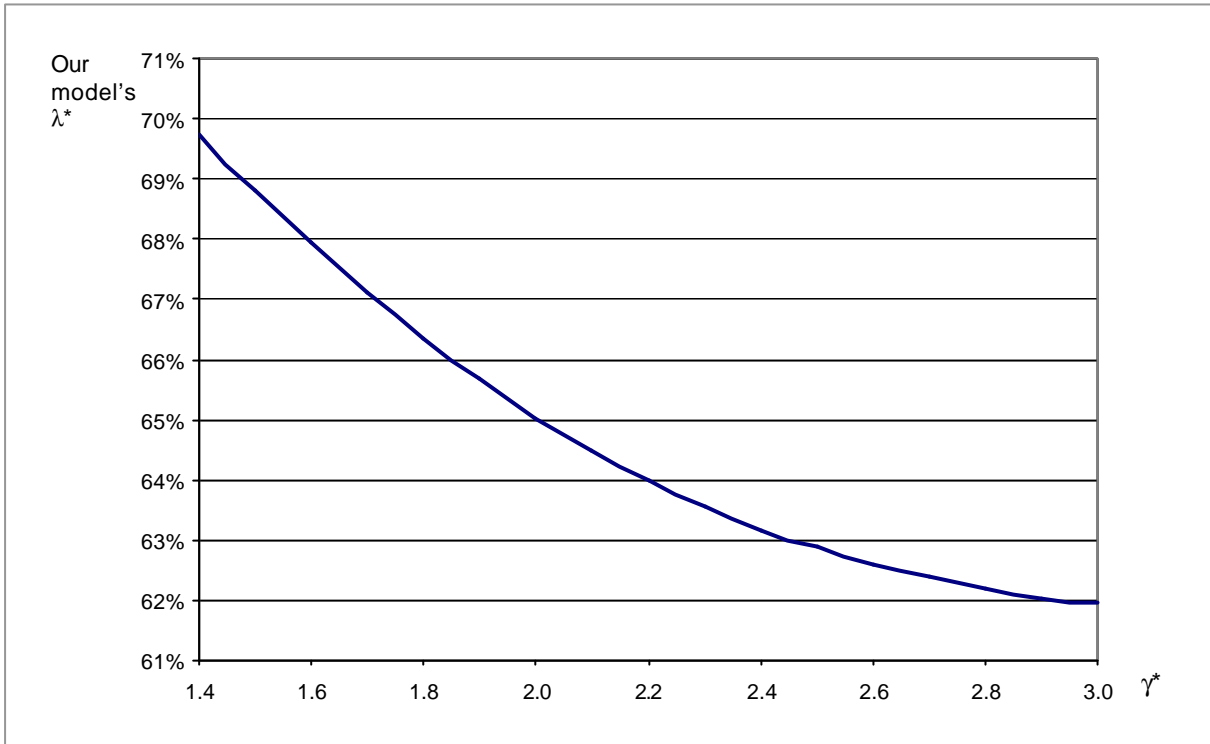
In fact, and as shown in Graph No. 1, it is possible to find a value for γ^* that guarantees a better fit between observed and predicted dollarization¹⁴. In terms of the discussion in the previous section, this implies that the corrected relative volatility of inflation vis-à-vis real depreciation (as expressed in equation (18.)) proves to be a superior statistic than σ_π^2/σ_s^2 if we are to track the persistence of financial dollarization. These results support the claim that the missing ingredient in the basic model is the connection between credit risk and real depreciation surprises, being these surprises the ones that directly affect realized real returns for dollar assets.

Finally, and taking the Dec-2004 prediction as a baseline, Graph No. 2 presents the results of simulating an increase in the threshold real depreciation shock up the point where truncation over the inflation and real depreciation distribution is no longer relevant. As predicted in the preceding section, lowering the size the of “fear” leads to a reduction in the degree of underlying dollarization¹⁵.

¹⁴ It is worth noticing that the size of the “fear” required to minimize the difference between observed and predicted dollarization falls monotonically since 1999. In fact, it comes as no surprise the “fear” reached a peak around this year, since the combined effect of the Russian (Sept-1998) and Brazilian crises (Jan-1999) turned evident the existence of pervasive balance sheet effects in the Peruvian economy (precisely the type of credit risk that our model is intended to capture).

¹⁵ Increasing the threshold real depreciation shock that triggers default implies that all relevant second moments converge to the values considered in the basic model. Our model’s predictions, however, fail to converge to those of the basic model because our numerical simulation explicitly ruled out the possibility of short sales for every type of asset. The analytical solution to the basic model (given by equation (14.)), on the other hand, is the result of an unconstrained minimization problem.

Graph No. 2
Reducing the “Fear”



4 Conclusions and Avenues for Further Research

As shown in Ize (2005), the MVP is the only stable equilibrium under risk aversion. Based on this result, a significant strand of the debate concerned with the driving forces behind financial dollarization has focused on analyzing the determinants of the relative variability of inflation vis-à-vis that of real depreciation. In particular, the discussion has focused on identifying the diverse features of monetary policy that can affect the distributions from where inflation and real depreciation shocks are drawn.

At the core of this discussion we can find two leading (and opposing) forces: inflation targeting and fear of floating. The former, by effectively reducing inflation variability, should help mitigate financial dollarization. The latter, on the other hand, by making dollar assets more attractive as hedging instruments, would act in the opposite direction.

This analysis has contributed in the identification of those factors that can end up altering the perceived distributions from where inflation and real exchange rate surprises can be drawn and, thus, the relative volatility of peso and dollar returns. In particular, the key ingredient of our model is the existence of local assets which face a credit risk that is directly linked to the shock that determines real returns for dollar denominated assets: unanticipated shifts in the real exchange rate. As shown, this ingredient can end up altering the perceived relative volatility of peso and dollar assets in a way that fuels financial dollarization (by increasing the relative hedging opportunities offered by the latter).

This analysis has taken the degree of credit risk as given. In fact, we have assumed that risk averse depositors choose the optimal composition of their asset portfolio trying to minimize the variance of its returns, for a given probability of default which, in turn, depends on the realization of a sufficiently large real depreciation shock. Therefore, one possible extension to our model calls for endogenizing the threshold real depreciation shock that triggers default. One obvious candidate if we are to explain the size of this critical shock is the degree of financial dollarization. In fact, we can expect smaller depreciations to trigger default in highly dollarized economies as a result of the existence of pervasive currency mismatches. If this is the case, underlying dollarization as predicted by the MVP could be portrayed as a self-reinforcing phenomenon.

In fact, and as in Broda and Levy Yeyati's (2003) setting, larger currency mismatches (driven by a high and persistent asset dollarization) would increase the size of the "fear" which will, in turn and according to our model's results, fuel even more the degree of asset dollarization up to the point where the "fear" is maximized (default can be triggered by a very small real depreciation shock). On the other hand, any exogenous fall in the degree of financial dollarization could also be self-reinforced: the reduction in the size of the "fear" would fuel financial de-dollarization up to the point where the "fear" is minimized, and we will eventually end up with a degree of financial dollarization consistent with the predictions of the basic CAPM model.

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Appendix A

The general formula for the variance decomposition in a joint distribution:

$$\text{Var}(y) = E_x \left[\text{Var} \left[y | x \right] \right] + \text{Var}_x \left[E \left[y | x \right] \right] \quad (\text{A1})$$

states that the variance of y decomposes into mean of the conditional variance plus the variance of the conditional mean. In terms of the stochastic variables considered in this analysis, this amounts to:

$$\begin{aligned} \text{Var}(r^i) &= \Pr(\mu_s < \delta^*) \text{Var} \left[r^i | \mu_s < \delta^* \right] + \Pr(\mu_s \geq \delta^*) \text{Var} \left[r^i | \mu_s \geq \delta^* \right] + \dots \\ &\dots + \Pr(\mu_s < \delta^*) \left\{ E \left[r^i | \mu_s < \delta^* \right] - E \left[r^i \right] \right\}^2 + \Pr(\mu_s \geq \delta^*) \left\{ E \left[r^i | \mu_s \geq \delta^* \right] - E \left[r^i \right] \right\}^2 \end{aligned} \quad (\text{A2})$$

$$\begin{aligned} \text{Cov}(r^i, r^j) &= \Pr(\mu_s < \delta^*) \text{Cov} \left[r^i, r^j | \mu_s < \delta^* \right] + \Pr(\mu_s \geq \delta^*) \text{Cov} \left[r^i, r^j | \mu_s \geq \delta^* \right] + \dots \\ &\dots + \Pr(\mu_s < \delta^*) \left\{ E \left[r^i | \mu_s < \delta^* \right] - E \left[r^i \right] \right\} \left\{ E \left[r^j | \mu_s < \delta^* \right] - E \left[r^j \right] \right\} + \dots \\ &\dots + \Pr(\mu_s \geq \delta^*) \left\{ E \left[r^i | \mu_s \geq \delta^* \right] - E \left[r^i \right] \right\} \left\{ E \left[r^j | \mu_s \geq \delta^* \right] - E \left[r^j \right] \right\} \end{aligned} \quad (\text{A3})$$

from where it is clear that (A2) is a special case of (A3) when $i = j$. Given the assumption of normality for real depreciation surprises, and if we let $\gamma^* = \delta^*/\sigma_s$, we can define:

$$\begin{aligned} \Pr(\mu_s < \delta^*) &= \Phi(\gamma^*) \\ \Pr(\mu_s \geq \delta^*) &= 1 - \Phi(\gamma^*) \end{aligned} \quad (\text{A4})$$

In order to build second moments for the three assets considered, we require expressions for means, variances and covariances conditioned over the real depreciation shock. From the expressions given in (2.), (3.) and (4.) in the main text, and since confiscation risk is uncorrelated with inflation and real depreciation surprises, it is clear that:

$$\begin{aligned} E \left[r^F | \mu_s < \delta^* \right] &= \bar{r}^F + E \left[\mu_s | \mu_s < \delta^* \right]; \quad E \left[r^F | \mu_s \geq \delta^* \right] = \alpha_F \left\{ \bar{r}^F + E \left[\mu_s | \mu_s \geq \delta^* \right] \right\} \\ \text{Var} \left[r^F | \mu_s < \delta^* \right] &= \text{Var} \left[\mu_s | \mu_s < \delta^* \right] + \sigma_c^2; \quad \text{Var} \left[r^F | \mu_s \geq \delta^* \right] = \alpha_F^2 \left\{ \text{Var} \left[\mu_s | \mu_s \geq \delta^* \right] + \sigma_c^2 \right\} \end{aligned} \quad (\text{A5})$$

$$\begin{aligned} E[r^H | \mu_s < \delta^*] &= \bar{r}^H - E[\mu_\pi | \mu_s < \delta^*]; \quad E[r^F | \mu_s \geq \delta^*] = \alpha_H \{ \bar{r}^H - E[\mu_\pi | \mu_s \geq \delta^*] \} \\ \text{Var}[r^H | \mu_s < \delta^*] &= \text{Var}[\mu_\pi | \mu_s < \delta^*] + \sigma_c^2; \quad \text{Var}[r^H | \mu_s \geq \delta^*] = \alpha_H^2 \{ \text{Var}[\mu_\pi | \mu_s \geq \delta^*] + \sigma_c^2 \} \end{aligned} \quad (\text{A6})$$

$$\text{Cov}[r^F, r^H | \mu_s < \delta^*] = -\text{Cov}[\mu_s, \mu_\pi | \mu_s < \delta^*] + \sigma_c^2; \quad (\text{A7})$$

$$\text{Cov}[r^F, r^H | \mu_s \geq \delta^*] = -\alpha^F \alpha^H \{ \text{Cov}[\mu_s, \mu_\pi | \mu_s \geq \delta^*] + \sigma_c^2 \}$$

$$\text{Cov}[r^F, r^C | \mu_s < \delta^*] = \text{Var}[\mu_s | \mu_s < \delta^*]; \quad (\text{A8})$$

$$\text{Cov}[r^F, r^C | \mu_s \geq \delta^*] = \alpha^F \text{Var}[\mu_s | \mu_s \geq \delta^*]$$

$$\text{Cov}[r^H, r^C | \mu_s < \delta^*] = -\text{Cov}[\mu_s, \mu_\pi | \mu_s < \delta^*]; \quad (\text{A9})$$

$$\text{Cov}[r^H, r^C | \mu_s \geq \delta^*] = -\alpha^H \text{Cov}[\mu_s, \mu_\pi | \mu_s \geq \delta^*]$$

Given the joint distribution assumed for inflation and real depreciation surprises:

$$\begin{bmatrix} \mu_s \\ \mu_\pi \end{bmatrix} \sim N \left[\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_s^2 & \sigma_{s\pi} \\ \sigma_{s\pi} & \sigma_\pi^2 \end{pmatrix} \right],$$

finding an expression for all required first and second moments amounts to the application of the general formulae for conditional means and variances in a normal bivariate (of the class typically used when analyzing truncation and incidental truncation). If we let $\rho = \sigma_{s\pi} / \sigma_s \sigma_\pi$, these are:

$$\begin{aligned} E[\mu_s | \text{truncation over } \mu_s] &= 0 + \sigma_s \lambda(\gamma^*) \\ \text{Var}[\mu_s | \text{truncation over } \mu_s] &= \sigma_s^2 [1 - \delta(\gamma^*)] \end{aligned} \quad (\text{A10})$$

$$\begin{aligned} E[\mu_\pi | \text{truncation over } \mu_s] &= 0 + \rho \sigma_\pi \lambda(\gamma^*) \\ \text{Var}[\mu_\pi | \text{truncation over } \mu_s] &= \sigma_\pi^2 [1 - \rho^2 \delta(\gamma^*)] \end{aligned} \quad (\text{A11})$$

$$\text{Cov}[\mu_s, \mu_\pi | \text{truncation over } \mu_s] = \frac{\sigma_{s\pi}}{\sigma_s^2} \text{Var}[\mu_s | \text{truncation over } \mu_s] = \sigma_{s\pi} [1 - \delta(\gamma^*)]$$

where:

$$\lambda(\gamma^*) = \begin{cases} -\frac{\phi(\gamma^*)}{\Phi(\gamma^*)} & \text{if truncation is } \mu_s < \delta^* \\ \frac{\phi(\gamma^*)}{1-\Phi(\gamma^*)} & \text{if truncation is } \mu_s \geq \delta^* \end{cases} \quad (\text{A12})$$

and $\delta(\gamma^*) = \lambda(\gamma^*)[\lambda(\gamma^*) - \gamma^*]$.

Equations (9.) and (10.) in the main text can be obtained by evaluating (A2) considering expressions given in (A5) plus (A10), and (A6) plus (A11), respectively. Finally, all three covariances (equations (11.), (12.), and (13.) in the main text) can be obtained by evaluating (A3) considering expressions given in (A7) to (A9), the general formula for the conditional covariance given in (A11), and the appropriate conditional means.